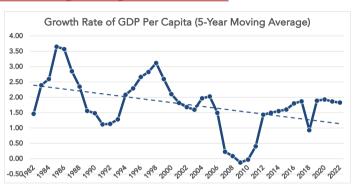
The Inequality-Growth Nexus: Evidence and Theory from Capital Goods Innovation

Furkan Kilic

University of Chicago Capital Theory

May 6, 2025

Growth rate of GDP per capita has decreased



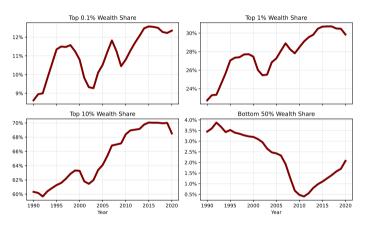
Source: St. Louis FED, FRED Database

Large literature on declining productivity and dynamism

- Akcigit and Ates (JPE, 2023): Declining knowledge diffusion
- Autor et al (QJE, 2020): Rise of superstar firms and market concentration
- Liu, Mian And Sufi (ECTA, 2022): Declining interest rates

and several others...

Personal wealth inequality has risen since past decades



Source: FED, Distributional Financial Accounts (DFA)

Another large literature on wealth and income inequality

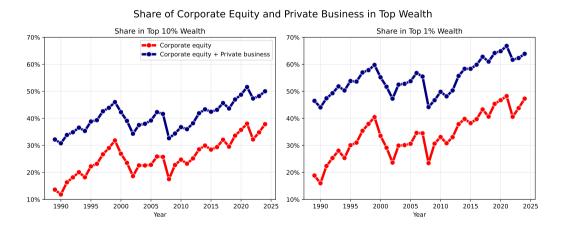
- Piketty and Saez (QJE, 2003): Income inequality
- Saez and Zucman (QJE, 2016): Wealth inequality
- Kuhn, Schularick and Steins (JPE, 2020): Role of business equity in driving wealth inequality
- **Piketty (2014):** *r* − *g* argument (see also Jones (JEP, 2015))
- Moll, Rachel and Restrepo (ECTA, 2022): Automation

and several others...



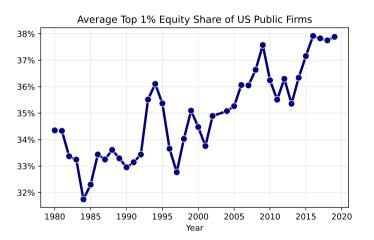
Equity and business share in top wealth rises

Kuhn, Schularick and Steins (JPE, 2020): Compositional diff. in portfolio across the wealth dist.



Source: FED, Distributional Financial Accounts (DFA)

Inequality across public firms increased



Source: Compustat.

Notes: Equity is defined as total asset minus total liabilities. Top 1% share of equity of each 4-digit NAICS industry x year is aggregated to year level with industry's total sale.

How does it affect inequality?

What is the role of technology?

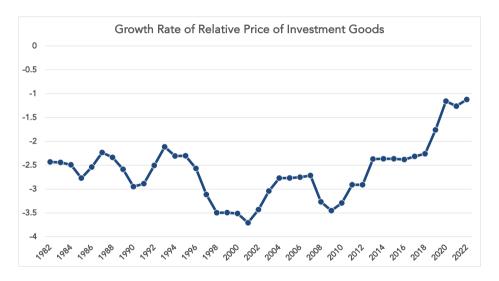
Effect of technology on inequality across firms

$$\log y_{s,t+m} - \log y_{s,t} = \beta x_{s,t}(n) + \gamma \log y_{s,t} + \alpha_s + \delta_t + \varepsilon_{s,t} \quad \text{for } m = 3 \text{ and } n \in \{0,3,5\}$$

Tech Shock - Current Year	$\Delta_3 \log(\text{Equity p90-p10 Ratio})$			$\Delta_3 \log(\text{Equity p99-p90 Ratio})$			$\Delta_3 \log(\text{Equity Top 1\% Share})$			Δ_3 log(Equity Top 10% Share)		
	-9.722 (10.386)			-5.014 (7.498)			-3.768 (4.051)			0.234 (1.512)		
Tech Shock - Past 3 Years		-38.034 (24.214)			-26.879* (14.139)			-20.292** (8.832)			-3.657 (4.018)	
Tech Shock - Past 5 Years		(=====,	-103.869*** (35.665)		(,	-36.157* (20.227)		(=====)	-15.224 (16.666)		(=====,	-8.357* (4.971)
Log(90-10 Ratio)	-0.283*** (0.015)	-0.301*** (0.018)	-0.310*** (0.019)			(,			(,
Log(99-90 Ratio)	(******)	((-0.283*** (0.022)	-0.298*** (0.020)	-0.296*** (0.021)						
Log(Top 1% Share)				, ,	, ,	, ,	-0.249*** (0.023)	-0.247*** (0.021)	-0.273*** (0.025)			
Log(Top 10% Share)							(0.020)	(0.021)	(0.020)	-0.245*** (0.035)	-0.255*** (0.040)	-0.273*** (0.031)
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1034	798	644	1034	798	644	1034	798	644	1034	798	644
N. of groups	90	87	83	90	87	83	90	87	83	90	87	83
R^2	0.434	0.462	0.512	0.447	0.433	0.411	0.387	0.372	0.401	0.337	0.350	0.417
Residual Std. Error	0.209	0.216	0.223	0.119	0.107	0.097	0.053	0.047	0.048	0.022	0.021	0.022
F Statistic	356.371***	299.271***	288.274***	375.437***	266.885***	191.583***	292.717***	206.831***	184.263***	235.682***	188.284***	196.580**

 $^*p{<}0.1;\,^{**}p{<}0.05;\,^{***}p{<}0.01$

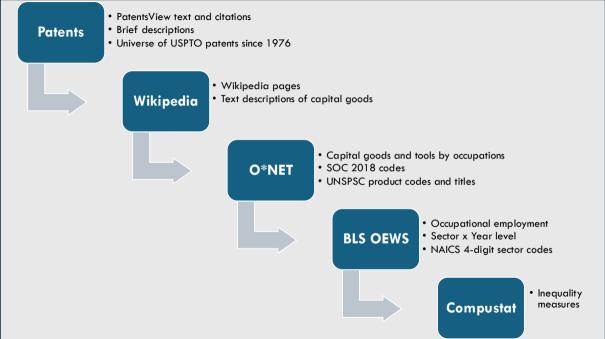
Decline in investment good prices has slowed down



Source: St. Louis FED, FRED Database

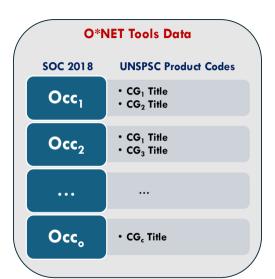
Quantifying Technology Shocks

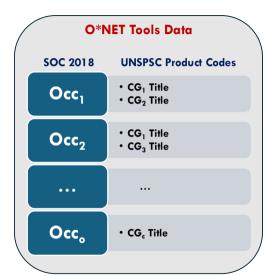
Data

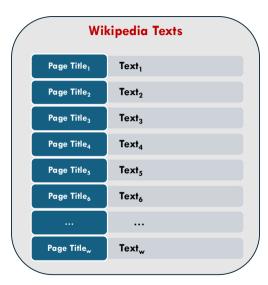


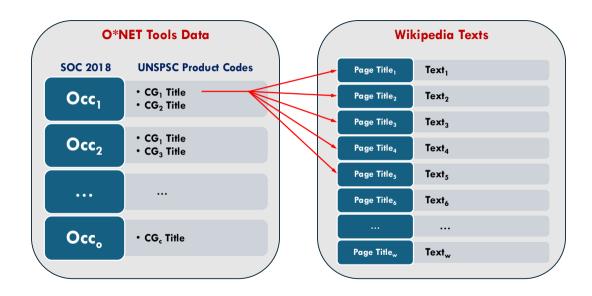
Capital Goods and Text Descriptions

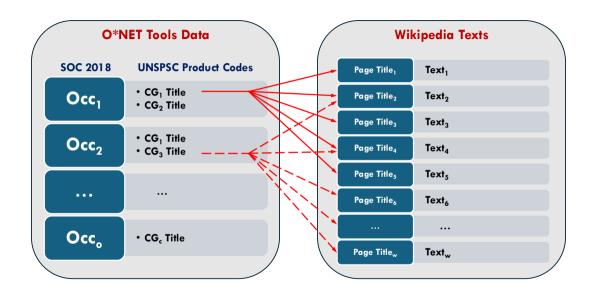
Quantifying Technology Shocks











Machine Learning, Embedding Models and Text Similarity

Quantifying Technology Shocks

Machine Learning Recap: Embedding Models

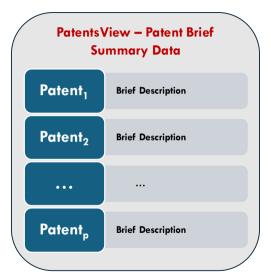
An embedding model is any function that maps words, sentences or documents into a vector space

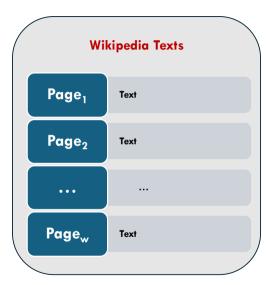
$$F(\text{text}) = \vec{v}$$

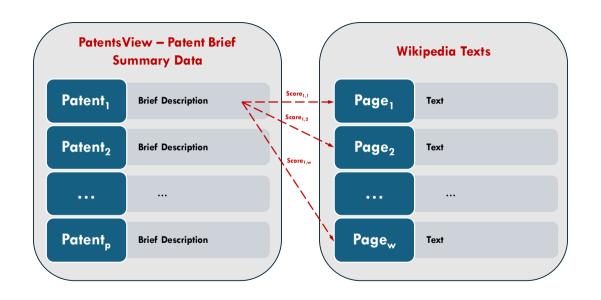
⇒ The goal is to map semantically similar texts into closer points on the vector space

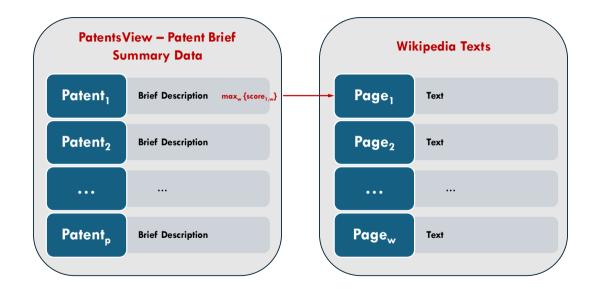
- ► "Attention Is All You Need" (Vaswani et al., 2017) transformed embedding models with self-attention mechanism giving rise to recent AI chat tools like ChatGPT
- ▶ ModernBERT is a recent transformer-based embedding model that turns words and sentences into numerical vectors reflecting their meaning in context
- ▶ I use ModernBERT ML model to map **Patent** and **Wikipedia** texts into 1024 dimensional vectors to perform **text-similarity analysis**

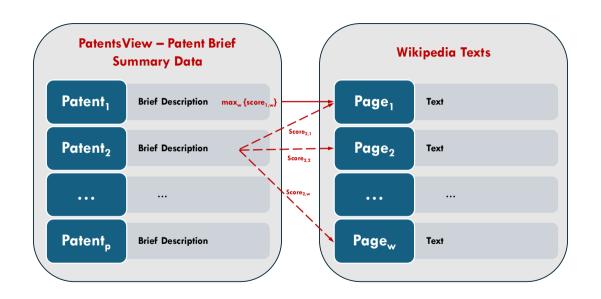
$$score_{p,w} = \frac{\vec{v}_p \cdot \vec{v}_w}{\|\vec{v}_p\| \|\vec{v}_w\|} \in [-1,1]$$

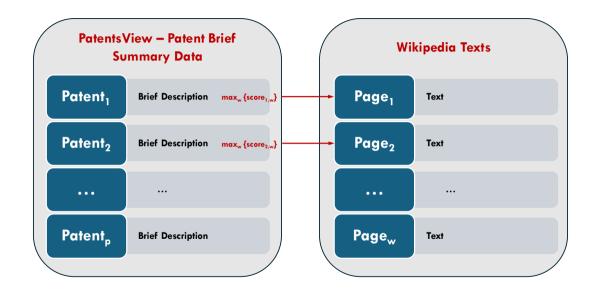


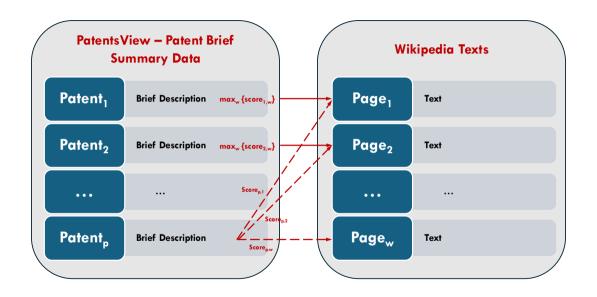


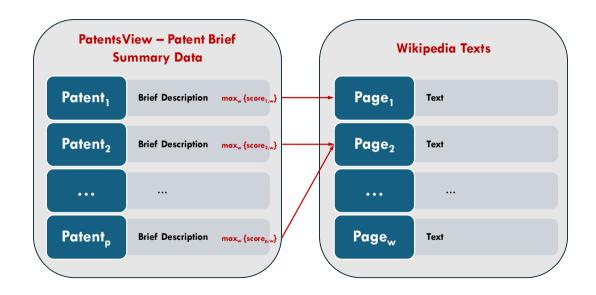




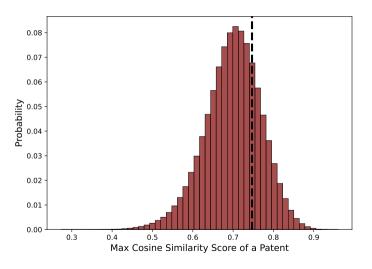






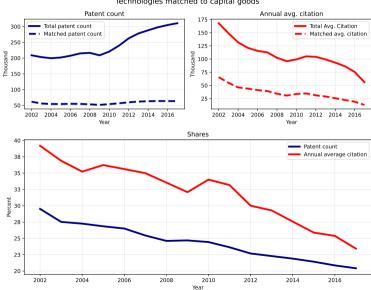


Distribution of top similarity scores



Capital augmenting technical change slows down





Construction of technology shocks

- Let $z_{c,t}$ be the technology flow measure for capital good c at year t
- I measure this object by either of the following: Patent count or Average annual forward citations
- Let $Z_{c,t}$ denote the stock of technology
- Average technology flow rate to capital good c in the past n = 0, 1, ... years is defined as

$$\tilde{z}_{c,t}(n) := \frac{\left(\sum_{\tau=0}^{n} z_{c,t-\tau}\right)/n}{\left(\sum_{\tau=0}^{n} Z_{c,t-\tau}\right)/n}$$

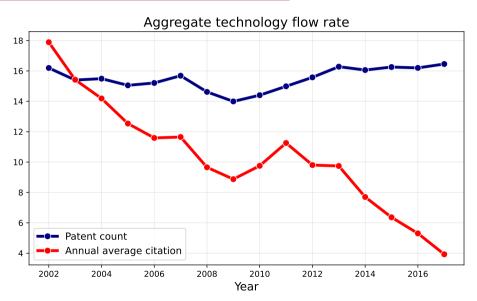
• Let $\tilde{n}_{o,s,t}$ denote employment of occupation o in sector s at time t (BLS-OEWS data). Employment associated with capital good c is defined as follows

$$n_{c,s,t} := \sum_{o \in O(s,t)} \mathbf{1} \left\{ o \in O(c) \right\} \tilde{n}_{o,s,t}$$

Finally, sector level technology shocks are defined as

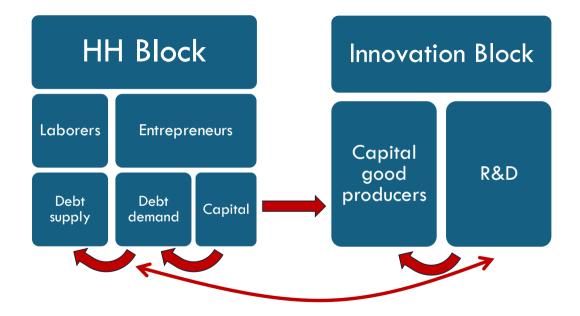
$$x_{s,t}(n) := \sum_{c} \frac{n_{c,s,t}}{\sum_{c} n_{c,s,t}} \cdot \tilde{z}_{c,t}(n)$$

Employment weighted aggregate technology shocks



Theory

A model of inequality and growth



Laborers

Denoting calendar time with t, a laborer i who had the last dissipation shock at \underline{t} solves

$$\mathcal{V}_{i,\underline{t}}(0) = \max_{\left\{C_{i,\underline{t}+s}\right\}_{s\geq 0}} \mathbb{E}\left[\int_{0}^{S} e^{-\varrho s} \ln C_{i,\underline{t}+s} ds + e^{-\varrho S} \mathcal{V}_{i,\underline{t}+S}(0)\right]
\text{s.t. } \dot{B}_{i,t} = r_{t} B_{i,t} + W_{t} + \frac{\Pi_{t}}{L} + \frac{T_{t}}{L} - C_{i,t}, \quad t \geq \underline{t}
B_{i,t} = 0 \quad \text{and} \quad B_{i,S} = 0$$
(1)

Notation:

- $S \geq \underline{t}$: Time at which next dissipation shock arrives with rate θ
- $B_{i,t}$: Risk-free bond holdings at time t
- *r*_t Risk-free interest rate
- *W_t* Wage rate
- Π_t : Profits (dividends) $\implies \frac{\Pi_t}{L}$: profits per person
- T_t : Lump-sum transfers

Entrepreneurs - Investment

Portfolio choice problem: Risky capital good ($\mathcal{K}_{i,t}$) vs Risk-free bond ($B_{i,t}$):

$$\chi_{j,t} := \underbrace{Q_t K_{j,t}}_{K_{j,t}} + B_{j,t}$$

• Capital is rented to competitive final good producers with technology

$$Y_t = K_t^{\alpha} (M_t \cdot L)^{1-\alpha}$$

- R_t and Q_t denote the rental rate and price of capital, respectively $\left(g_t^Q := \dot{Q}_t/Q_t\right)$
- · Physical capital invested is subject to idiosyncratic productivity shocks and depreciation

$$\frac{dK_{j,t}}{K_{i,t}} = -\delta \cdot dt + \nu \cdot dW_{j,t}$$

Return on capital:
$$dR_{j,t}^k := \underbrace{(1-\tau)\frac{R_t}{Q_t}\cdot dt}_{\text{dividend yield}} + \underbrace{(-\delta+g_t^Q)\cdot dt + \nu\cdot d\mathcal{W}_{j,t}}_{\text{capital gain}} = \underbrace{\left((1-\tau)\frac{R_t}{Q_t} - \delta + g_t^Q\right)\cdot dt + \nu\cdot d\mathcal{W}_{j,t}}_{:=r^k}$$

Entrepreneurs

Denoting calendar time with t, an entrepreneur j who had the last dissipation shock at \underline{t} solves

$$\mathcal{V}_{j,\underline{t}}(0) = \max_{\left\{C_{j,\underline{t}+s},\kappa_{\underline{t}+s}\right\}_{s\geq 0}} \mathbb{E}\left[\int_{0}^{S} e^{-\varrho s} \ln C_{j,\underline{t}+s} ds + e^{-\varrho S} \mathcal{V}_{j,\underline{t}+S}(0)\right]$$
s.t.
$$d\chi_{j,t} = dR_{j,t}^{k} \cdot \mathcal{K}_{j,t} + r_{t} dt \cdot \left(\chi_{j,t} - \mathcal{K}_{j,t}\right) + \left(W_{t} + \frac{\Pi_{t}}{L} + \frac{T_{t}}{L}\right) \cdot dt - C_{j,t} \cdot dt, \quad t \geq \underline{t}$$

$$\mathcal{K}_{j,t} \geq 0$$

$$\chi_{j,t} = 0 \quad \text{and} \quad \chi_{j,S} = 0$$

$$(2)$$

where rate of return on capital equals

$$dR_{j,t}^k = r_t^k \cdot dt + \nu \cdot dW_{j,t}$$

$$r_t^k = (1 - \tau) \frac{R_t}{Q_t} - \delta + g_t^Q$$

Investment good producers

• New capital goods are produced by a competitive market from capital varieties $n \in [0,1]$ with **heterogeneous quality/productivity**

$$I_t = \exp\left[\int_0^1 \ln\left(a_{n,t}I_{n,t}\right) dn\right]$$

- $\rightarrow a_{n,t}$: productivity of capital good variety n
 - Incumbent firms produce 1 unit of capital variety from 1 unit of final good
- Aggregate capital stock K_t follows (idiosyncratic shocks wash out)

$$\dot{K}_t = -\delta K_t + I_t$$

Research and development

- Research is free to enter and directed to any variety *n*
- The cost of producing a rate of innovation of *z* equals (in terms of final good)

$$\frac{z}{\eta}Q_tI_t$$

- $\rightarrow \eta$ denotes research productivity
- → Cost of innovation is indexed by the size of investment market
 - Successful innovators improve the productivity of a variety with a step size of $\lambda > 1$

Successful innovation
$$\implies a_{n,t+dt} = \lambda a_{n,t}$$

• The innovator obtains the new blueprint for the variety allowing them the monopoly rights

Theory

Equilibrium

Equilibrium

Equilibrium is defined as follows

- Laborers choose consumption $C_{i,t}$ and bond holdings $B_{i,t}$ to maximize life-time utility (1)
- Entrepreneurs choose consumption $C_{j,t}$, capital investment $K_{j,t}$ and bond holdings $B_{j,t}$ to maximize life-time utility (2)
- Perfectly competitive final good and investment good producers maximize profits by taking input prices as given
- Monopolistically competitive capital variety producers maximize profits
- Researchers maximize profits + Free entry condition
- Bond and capital markets, and all good markets clear

Balanced growth path (BGP) equilibrium

In this equilibrium:

- 1. All relevant growth rates are constants: g, g^Q and g^K
- 2. Risk-free interest rate *r* is constant
- 3. The growth rate of the capital stock in final goods equal the growth rate of output

$$\frac{(Q_t \dot{K}_t)}{Q_t K_t} = \frac{\dot{Y}_t}{Y_t} \implies g^Q + g^K = g$$

- ⇒ In other words: value of capital stock in terms of final good to output ratio is constant
 - **4**. All other remaining aggregate variables growth at rate *g*

NOTE: All aggregate variables are normalized by the capital stock Q_tK_t and denoted by **lowercase letters**.

Theory

Innovation Block

Investment good producers

$$\max_{I_{n,t}} Q_t \exp \left[\int_0^1 \ln (a_{n,t} I_{n,t}) dn \right] - \int_0^1 q_{n,t} I_{n,t} dn \implies I_{n,t} = q_{n,t}^{-1} Q_t I_t$$

- Incumbent producers charge limit price: $q_{n,t} = \lambda \implies \Pi_{n,t} = (1 \lambda^{-1})Q_tI_t$
- Price of capital goods equals

$$Q_t = \lambda A_t^{-1}$$
 where $A_t := \exp\left[\int_0^1 \ln a_{n,t} dn\right]$

R&D and free entry into research

Taking creative destruction rate z_n given, the value of owning variety n satisfies HJB eq.

$$rV_{n,t} = (1 - \lambda^{-1})Q_tI_t - \tau_n V_{n,t} + \dot{V}_{n,t}$$

$$\implies$$
 We can show, $V_{n,t}=v_nQ_tI_t$ where $v_n=rac{1-\lambda^{-1}}{r+z_n-g^{Q\cdot I}}$

Free entry into research implies

$$\max_{\tilde{z}} \left\{ -\frac{\tilde{z}}{\eta} Q_t I_t dt + \tilde{z} dt \cdot v_n Q_t I_t \right\} = 0 \implies v_n = \frac{1}{\eta}$$

We can solve for rate of creative destruction/innovation as

$$z_n = g^{Q \cdot I} - r + \eta (1 - \lambda^{-1}) \quad \forall n$$

Growth rates

Growth rate of capital price is negatively associated with innovation:

$$g^A := \frac{\dot{A}_t}{A_t} = z \cdot \ln \lambda \implies g^Q = -z \cdot \ln \lambda$$

Capital accumulation $\dot{K}_t/K_t = -\delta + I_t/K_t \implies g^I = g^K$ and $g^{Q \cdot I} = g^{Q \cdot K} = g$

$$\implies z = g - r + \eta (1 - \lambda^{-1})$$

Finally, using final good production function $\implies g = \alpha g^K + (1 - \alpha)g^Z$ and $g = g^Q + g^K$

$$\implies \left| \begin{array}{l} g^K = \frac{1}{1-\alpha} z \ln \lambda + g^M \\ g = \frac{\alpha}{1-\alpha} z \ln \lambda + g^M \end{array} \right|$$

Theory

Households, capital accumulation and wealth inequality

Laborers

Denote total (effective) wealth of the agent *i* as the sum of financial and human capital wealth:

$$X_{i,t} := B_{i,t} + \frac{H_t}{r-g}, \quad H_t := W_t + \frac{\Pi_t + T_t}{L}$$

 \implies Relevant state variable is $X_{i,t}$

We can show

$$\dot{X}_{i,t} = rX_{i,t} - C_{i,t}$$

and

$$C_{i,t} = \rho X_{i,t} \implies \dot{X}_{i,t} = (r - \rho) X_{i,t}$$

where

$$\rho := \varrho + \theta$$

Laborers - Debt supply

Define

$$\tilde{r} := r - \rho - g$$

 \implies Rate of return of individual normalized effective wealth is \tilde{r} :

$$\frac{\dot{x}_{i,t}}{x_{i,t}} = \tilde{r}$$

We can show aggregate bond holdings of laborers, B_t^L , equal (conditional on $\theta > \tilde{r}$)

$$B_t^L = \frac{\tilde{r}}{\theta - \tilde{r}} \frac{H_t}{\tilde{r} + \rho} (1 - \psi) L \qquad \Longrightarrow \qquad b^L = \frac{\tilde{r}}{\theta - \tilde{r}} \frac{h}{\tilde{r} + \rho} (1 - \psi) L$$

 \implies Long run debt supply increases with $ilde{r}$

Entrepreneurs

Denote total (effective) wealth of the agent *j* as the sum of financial and human capital wealth:

$$X_{j,t} := \mathcal{K}_{j,t} + B_{j,t} + \frac{H_t}{r - g}$$

 \implies Relevant state variable is $X_{j,t}$

We can show

$$dX_{j,t} = \left[r^{K}dt + \nu \cdot dW_{j,t}\right] \cdot \mathcal{K}_{j,t} + rdt \cdot \left(X_{j,t} - \mathcal{K}_{j,t}\right) - C_{j,t}dt$$

and, using $\rho := \varrho + \theta$,

$$C_{j,t} = \rho X_{j,t}$$
 and $\left[\frac{\mathcal{K}_{j,t}}{X_{j,t}} =: \omega = \frac{r^K - r}{\nu^2}\right] \Longrightarrow \frac{dX_{j,t}}{X_{j,t}} = \left[\underbrace{\omega r^K + (1 - \omega)r}_{j,t} - \rho\right] \cdot dt + \nu \omega \cdot d\mathcal{W}_{j,t}$

Entrepreneurs - Capital demand

Similar to laborers, define

$$\left\{ \begin{array}{c} \tilde{r}^K := r^K - \rho - g \\ \tilde{r}^E := r^E - \rho - g \end{array} \right\} \implies \tilde{r}^E = \omega \tilde{r}^K + (1 - \omega)\tilde{r} \quad \text{and} \quad \omega = \frac{\tilde{r}^K - \tilde{r}}{\nu^2}$$

 \implies Rate of return of individual normalized effective wealth is \tilde{r}^E :

$$\frac{dx_{j,t}}{x_{j,t}} = \tilde{r}^E \cdot dt + \omega \nu \cdot d\mathcal{W}_{j,t}$$

We can show that agg. capital and debt demand by ent., K_t^E and $-B_t^E$ equal (cond. on $\theta > \tilde{r}^E$)

$$\mathcal{K}_{t}^{E} = \frac{\omega \theta}{\theta - \tilde{r}^{E}} \frac{H_{t}}{\tilde{r} + \rho} \psi L \qquad \Longrightarrow \qquad \kappa^{E} = \frac{\omega \theta}{\theta - \tilde{r}^{E}} \frac{h}{\tilde{r} + \rho} \psi L$$
$$-B_{t}^{E} = \frac{\omega \theta - \tilde{r}^{E}}{\theta - \tilde{r}^{E}} \frac{H_{t}}{\tilde{r} + \rho} \psi L \qquad \Longrightarrow \qquad -b^{E} = \kappa^{E} - \frac{\tilde{r}^{E}}{\theta - \tilde{r}^{E}} \frac{h}{\tilde{r} + \rho} \psi L$$

Theory

Joining all rates together: Returns and innovation

Tilde returns

Return on capital

$$\omega$$
: Leverage

Return on ent. wealth

Return on laborer wealth

Return on laborer wealth

 $\tilde{r}^E = \omega \tilde{r}^K + (1 - \omega)\tilde{r}$ where $\omega = \frac{\tilde{r}^K - \tilde{r}}{v^2} \ge 0$
 $\Rightarrow \qquad \tilde{r}^E - \tilde{r} = \omega^2 v^2 = \left(\tilde{r}^K - \tilde{r}\right)^2 v^2$

Return on capital: \tilde{r}^K

$$\max K_t^{\alpha} (M_t L_t)^{1-\alpha} - R_t K_t - W_t L_t$$

gives rise usual factor price equations

$$R_t = \frac{\alpha Y_t}{K_t}$$
 and $W_t = \frac{(1-\alpha)Y_t}{L}$

Dividing these equations by Q_tK_t , we have

$$\frac{R_t}{Q_t} = \alpha y$$
 and $w = \frac{(1-\alpha)y}{L}$

where $y := \frac{Y_t}{O(K_t)}$ denotes output-capital ratio.

Return on capital: \tilde{r}^K

We defined

$$r^K := \mathbb{E}\left[dR_{j,t}^K\right] = (1-\tau)\frac{R_t}{Q_t} - \delta + g^Q \implies r^K = (1-\tau)\alpha y - \delta + g^Q$$

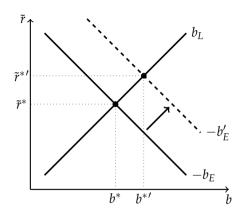
Using definition $\tilde{r}^K := r^K - \rho - g$ and growth rate expressions for g and g^Q , we show:

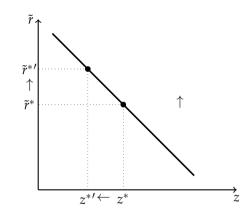
$$\boxed{ \vec{r}^{K} = (1 - \tau)\alpha y - \delta - \rho - \frac{1}{1 - \alpha} \ln \lambda \cdot z - g^{M} }$$

Two opposing effects of higher innovation z on capital return \tilde{r}^{K} :

- 1. Negative effect of creative destruction via capital gains g^Q
- 2. Positive effect of higher capital productivity via dividend yield and y

Feedback loop back to innovation: *z*

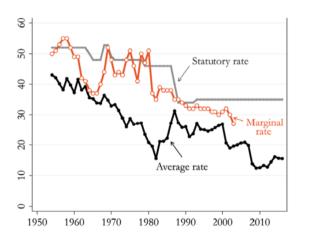




$$z = -\tilde{r} - \rho + \eta \left(1 - \lambda^{-1} \right)$$

Comparative Statics

Corporate tax rates in the US decline over time



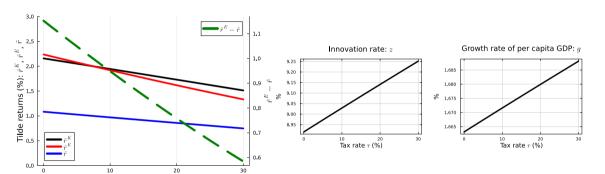
- Consider a permanent decline in τ
- Shifts $\tilde{r}^K \uparrow$
- Higher capital demand $\implies \omega \uparrow$
- Inequality measures \uparrow (both $\tilde{r}^E \uparrow$ and $\tilde{r}^E \tilde{r} \uparrow$)
- $\tilde{r} \uparrow \Longrightarrow z \downarrow$
- Starts over a second loop if negative effect dominates $z \downarrow \Longrightarrow \tilde{r}^K \uparrow \dots$

Source: Kaymak and Schott (ECTA, 2023)

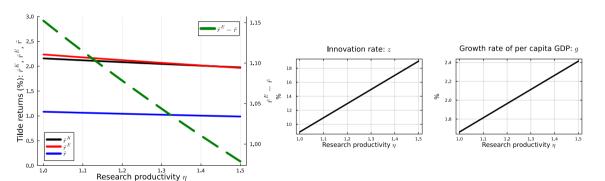
Effect of corporate tax rates: τ

20

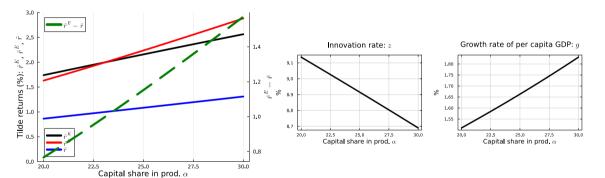
Tax rate τ (%)



Effect of research productivity: η



Effect of automation: α



Conclusion and next steps

In this paper:

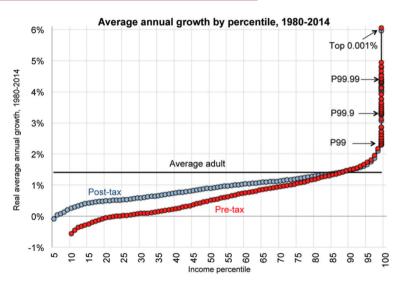
- I compile new data using machine learning tools on capital augmenting technological change
- I show that within-sector across-firm inequality is negatively associated with capital good innovations
- Also I show that aggregate innovation flow to capital goods decline over time
- Motivated by these facts, I build an endogenous growth model in which innovation and wealth inequality are jointly determined
- Declining corporate tax rates decreases growth whereas increasing inequality

Next steps:

- Exploiting regional variation in corporate tax rates to causally estimate the effect of innovation on firm inequality
- I may switch to ORBIS data as Compustat yields limited results

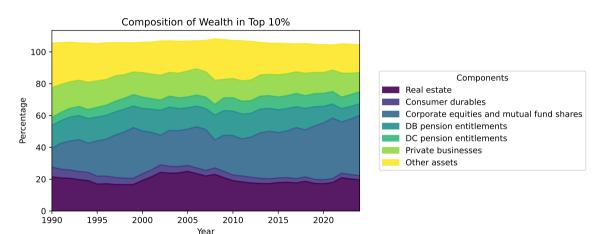
Appendix

Similar trends in income inequality •Back

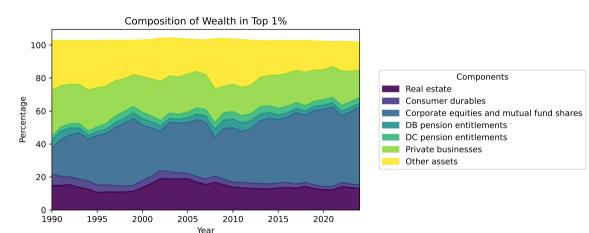


Source: Piketty, Saez and Zucman (QJE, 2018): Distributional National Accounts: Methods and Estimates for the United

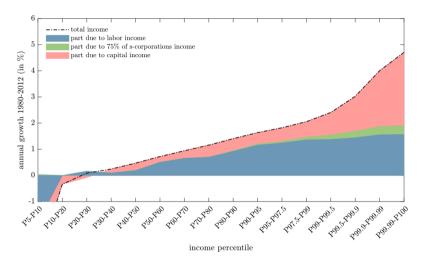
Share of equity in top wealth rises •Back



Share of equity in top wealth rises •Back

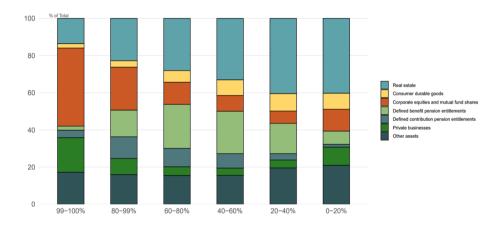


Capital's share in income growth •Back



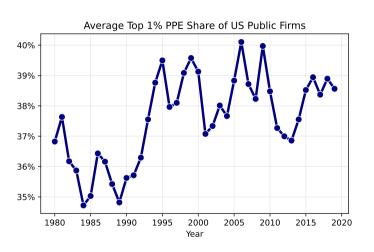
Source: Moll, Rachel and Restrepo (ECTA, 2022): Uneven Growth. Data from Piketty, Saez and Zucman (QJE, 2018) Distributional National Accounts.

Assets by income percentile in 2019 •Back



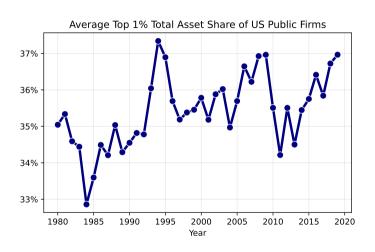
Source: FEDS Notes. https://www.federalreserve.gov/econres/notes/feds-notes/inequality-and-financial-sector-vulnerability 20240419.html (Accessed May 2025)

Inequality across public firms increased •Back



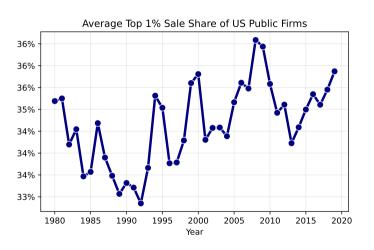
Source: Compustat.

Inequality across public firms increased •Back



Source: Compustat.

Inequality across public firms increased •Back



Source: Compustat.