Spatial Allocation of Inventors, Knowledge Diffusion and Growth*

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Abstract

Where does innovation truly thrive? Inventive activity in the US is strikingly concentrated in a handful of hubs. This raises compelling questions: Does further agglomeration drive innovation, or could a more dispersed approach better leverage regional spillovers? To investigate, I exploit variation in patent citation lags across US states and develop a novel endogenous growth model with mobile inventors and workers. The model integrates an exogenous knowledge network that facilitates the dynamic exchange of ideas—laying the foundation for future inventions—between locations, revealing that inventors do not internalize how their location choice influences broader knowledge diffusion. These knowledge spillovers call for a targeted, place-based R&D subsidy to unlock latent innovation potential. Calibrating the model to data on inventor and worker allocations—and estimating the knowledge diffusion network from patent citations—I find that optimal policy would further concentrate inventors in established hubs, enhancing welfare by 1.8 percent in consumption-equivalent terms and boosting the economy's long-run growth rate by 0.14 percentage points.

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1 Introduction

Innovation in the United States is strikingly concentrated in a few locations such as Silicon Valley in California, Route 128 in Massachusetts, and Seattle in Washington. In 2005, the top ten states produced 65 percent of all patents and hosted 64 percent of inventors, while accounting for only 47 percent of the population. The excessive concentration of innovative activity is pervasive, and noted in several studies (Buzard et al., 2020; Buzard and Carlino, 2013; Moretti, 2021), and it is not only specific to the US economy.[1] For example, Carrincazeaux et al. (2001) shows that six regions in France account for 75 percent of corporate R&D workers while their share of production workers is only 45 percent.[2]

The spatial allocation of inventors is crucial for the overall growth trajectory of an economy. Main reason can be traced back to Marshall (2009) in his treatment that localities benefit from knowledge spillovers within regions. Types of spillovers that are important for the creation of new inventions are in intertemporal nature, and can be traced by patent citations. Inventors build on top of the shoulders of the past when they develop new ideas. If knowledge diffusion between innovation centers is not frictionless, then inventors' location choices, or the accumulation of innovative activity in certain regions, would have nontrivial consequences on the technological development of the overall economy.

In this paper, I study intertemporal knowledge spillovers and spatial allocation of inventors across locations in the US, and its consequences on the long run growth of the economy. Firstly, I document signification variation in the spatial allocation of inventors relative to workers across states. Inventors relocate intensely to a few number of states that are usually home to innovation hubs. What is the effect of their location choice on the innovation process and the creation of ideas? I approach this question from the angle of intertemporal knowledge spillovers between regions. That is, rather than focusing only on within-location agglomeration externalities, I study knowledge flows across all regions, which are proxied by patent citations. Patent citations provide many advantages in this regard, as Jaffe et al. (1993) and Jaffe et al. (2000) argue, they can be interpreted as paper trails of knowledge spillovers between inventors, although inventors might live very far away from each other in space.

Knowledge spillovers, in the specific context of innovation and R&D, are mostly in intertemporal nature, finding its meaning in the famous phrase, new ideas are build on top of the shoulders of the past giants. Current inventors build on past inventions when they create new ideas, and they cite the ideas from which they benefit the most (Jaffe et al. (2000)). Analysis of patent citations data reveals that patent citations are also spatially concentrated. In particular, states like California, Massachusetts, Connecticut are the most cited states in the US. However, I argue that this observation alone cannot be interpreted as the importance of these states being the sole origins of idea creation hubs. Instead, I investigate patent citation lags between locations, and find significant variation across state-pairs, which suggests that some states are better connected to each other in the sense that they tend to cite

[1]See Carlino and Kerr (2015) for an extensive review of the literature.

[2]It must be noted that innovation hubs might move between regions over time, although this process is very slow. Lamoreaux et al. (2004) shows that Cleveland, Ohio, demonstrated similar characteristics as a dominant innovation hub with its sizable angel financing and startup incubators between 1870 and 1920. Similarly, Klepper (2010) compares Detroit's automobile manufacturing industry in the early twentieth century with the semiconductor industry in Silicon Valley today, noting the similarities between the two leading the way for excessive clustering.

each other relatively quicker than other state pairs. This variation in patent citation lags identifies the strength of connections between locations in terms of knowledge flows.

I study knowledge spillovers because of several reasons. In endogenous growth literature, intertemporal knowledge spillovers are identified as the main reason for the justification of R&D subsidy policies (Acemoglu et al., 2018; Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990). When inventors invent their ideas and develop new techniques, they do not internalize their effect on the creation of future ideas. This positive externality creates an incentive for the policy maker in favor of R&D subsidies. However, if knowledge spillovers vary across locations and are imperfect, this raises the question of whether uniform R&D subsidies are appropriate, or whether place-based subsidies should be designed instead—and if so, how such policies can be structured optimally.

Answering this normative question requires a theory of intertemporal knowledge spillovers between locations, inventor migration, and innovation as the engine of economic growth. I build a novel spatial economic growth model with endogenous inventor and worker migration choices, and heterogeneous knowledge linkages across locations. In the model, inventors and workers who hold idiosyncratic preferences for locations can move between regions freely albeit subject to a simple timing friction. Their objective is to maximize their life-time utility. Production side of the model is intentionally kept simple. In equilibrium, production workers earn the same wage income in all locations, thus their migration decision identifies location specific characteristics that are also common to inventors when they relocate. These characteristics are called amenities in the model. On top of amenity differences, locations in the model are heterogeneous in their fundamental research productivity. The idea is that some locations provide more resources for the R&D process such as the presence of leading universities like Stanford in California and MIT in Massachusetts, or certain institutions supporting innovation and dynamism.[3] These characteristics are unobserved, and they are estimated by matching the concentrated inventor allocation across US states. Another dimension by which locations are differentiated is their connectedness to the rest of the economy in terms of idea flows. In the model, past ideas invented in locations spill over to the rest over time, and they form the endogenous idea stock over which inventors in destination locations build on top when they perform R&D. The diffusion process is subject to frictions in that past ideas diffuse to other regions with a random time lag, where average time lag is specific to the location-pair, and it does not have to be symmetric. Inventors benefit both from location's fundamental (exogenous) resources and endogenous idea accumulation. All else equal, inventors that are located in states to which idea inflows are faster would be more productive in R&D, and they would earn higher wages. In the model, this is the source of externality of inventor location choice on the rest of the economy.

[3]See Jaffe (1989) for an early account on knowledge spillovers from research conducted in universities to firms located nearby. See also Nicholas and Lee (2013) for the special role of Stanford University in the development of Silicon Valley as an innovation hub. Kantor and Whalley (2014) find significant local spillovers from university activity identified from the interaction between university endowments and stock market shocks over time. These spillovers are larger when firms become closer to universities in the technology space. In addition to the role of universities, cities may also be heterogeneous in their entrepreneurship culture, and how failure is perceived among entrepreneurs. Landier (2005) provides a formal model in which two equilibria with different levels of entrepreneurship can emerge as a result of endogenous cost of failure for entrepreneurs.

The model also addresses endogenous location choices of firms who are employers of inventors. Potential entrants in the model are mobile across locations, and in equilibrium, they move to the location that provide the highest discounted future profits. Firms that enter to the market has a single R&D lab in a location in which inventors are employed, while they are indifferent across locations for the place of their production. In equilibrium, more research productive locations are home to a higher number of innovative firms who demand inventors more. Thus, the model simultaneously explains the presence of high volume of innovative firms in innovation hubs along with high number of inventors relocated there.

I quantify the model to recover location specific amenities and exogenous research productivities from the observed allocation of production workers and inventors. The knowledge network, which is the matrix of diffusion rates across state-pairs are estimated from patent citation lags in line with the idea accumulation process in the model. Estimation of the network reveals that (i) there is substantial heterogeneity in connectedness between locations, (ii) within-location spillovers are the strongest, and (iii) estimated idea diffusion rates increase with physical proximity and academic citation flows. After estimating the parameters, I test the model fit for a set of untargeted moments. This exercise suggests that the model performs well in explaining these moments.

I report the estimation results in two steps. In the first step, I estimate the model assuming that the US economy is comprised of only ten states where most of the patents are produced. I compare estimation results for two versions of the model—with and without knowledge spillovers. Then, I proceed with the optimal policy and run several counterfactual exercises to understand the nature of the policy. The optimal policy calls for more concentration in inventor allocation in space, while allocating inventors to more central states in the knowledge network. In the second step, I estimate the model for all US states. The optimal place based R&D subsidy policy concentrates inventors mostly in Washington, California and Massachusetts, although the model is abstracted away from reduced form agglomeration spillovers. The reason is that these states are connected well with the rest of the economy, while they are also the most research productive states. The welfare of the society increases by 1.8 percent in consumption equivalent terms under the optimal policy. Most of the increase in welfare stems from an increased long-run growth rate of the economy with a 0.14 percentage points.

Literature. A large body of research documents the strong spatial concentration of inventive activity. Empirical work has highlighted the persistence of localized knowledge spillovers causing agglomeration. Ellison and Glaeser (1997) and Duranton and Puga (2004) provide microfoundations for urban agglomeration economies. Helsley and Strange (2002) shows that the availability of a dense network of input providers in urban areas facilitates innovation by making it less costly for firms, hence concentrating inventive activity in large cities even without the presence of knowledge spillovers. Gerlach et al. (2009) shows, in a model with labor pooling in innovation centers, firms undertake voluminous and more diversified portfolios of R&D. Contrary to these microfoundations without knowledge spillovers, another strand of the literature focuses on the exchange of ideas in cities stressing the role of knowledge spillovers in creating agglomeration economies (Crews, 2023;

Davis and Dingel, 2019; Glaeser, 1999). Moretti (2021) finds using patent data that inventors experience sizable productivity increase in patenting after moving to a city with a large cluster of inventors in the same field. In this paper, I take a more general approach to knowledge diffusion across space and focus on spillovers not only within locations but also across locations. In my model, knowledge spillovers increase R&D productivity of inventors, and induce agglomeration economies for two reasons. States where past knowledge is shared among local inventors relatively quickly attract inventors relatively more. Moreover, states that are better connected to states with high research activity benefit from spillovers disproportionately more causing inventor inflow.

Similar to the methodology in this paper, knowledge diffusion across space has been studied extensively using patent citations. Jaffe et al. (1993) first showed that patent citations are disproportionately local, which they interpreted as evidence of geographically localized knowledge spillovers. Although Krugman (1992) argued that knowledge spillovers leave no paper trail, Jaffe et al. (1993) provide a comprehensive account of the role of patent citations in tracing intertemporal spillovers. Thompson and Fox-Kean (2005) emphasize the importance of carefully designed control patents in measuring the degree of knowledge diffusion in reduced form settings. On the other hand, Singh (2005) confirms strong intraregional spillovers in a regression framework and highlights the role of inventor mobility and social networks in shaping diffusion patterns. While these papers mainly focus on estimating localized spillovers from patent citations, Peri (2005) provides estimates on the fraction of knowledge diffusing outside of the origin region in a global setting. The empirical strategy in this paper is closest to Peri (2005), Caballero and Jaffe (1993) and Cai et al. (2022) in deriving an estimating equation from a structural relationship between idea flows and resulting citation patterns. In my model, the knowledge diffusion network plays a central role in spatial allocation of inventors, and at the same time, the heterogeneous intensity of idea flows between state pairs results in a structural equation for patent citations which allows me to estimate the entire network.[4]

The growth and policy literature emphasized the role of intertemporal knowledge spillovers in justifying R&D support (Acemoglu et al., 2018; Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990). More recent work has studied R&D externalities at the firm and product level (Akcigit and Kerr, 2018; Bloom et al., 2013) and the design of optimal innovation policies (Akcigit et al., 2022). This work generally treats space as homogeneous, while my contribution is to integrate spatial heterogeneity into the design of R&D policy.

Finally, the literature on place-based policies provides a natural benchmark for my analysis. Kline and Moretti (2014) review the effects of local economic development programs, while Fajgelbaum et al. (2019) examine the spatial allocation of economic activity in response to heterogeneous taxation at the state level. Gaubert (2018) further link firm sorting and spatial inequality to policy, concluding that subsidizing small cities reduce aggregate welfare. In a recent paper, Gross and Sampat (2023) shows

[4]Knowledge diffusion across technology fields has also recently gained attention in the literature. Acemoglu et al. (2016) model the idea production in a downstream technology class as a function of stock of knowledge diffused from upstream classes, similar to input-output production networks. This input-output network is quantified from the ratio of citations between two technology classes to the number of patents generated. Liu and Ma (2021) takes this idea to a general equilibrium setting and analyzes the optimal allocation of R&D resources across technology fields.

that place-based R&D subsidies can create long-lasting agglomeration effects. In parallel, Desmet et al. (2018) develop a dynamic model of spatial development with realistic geography and knowledge flows but without creative destruction, and Cai et al. (2022) analyzes the three-way interaction between international trade, innovation, and diffusion along with input-output linkages across industries. While I abstract away from geography, trade and input-output network in production, I particularly focus on location specific R&D subsidy policy in a closed economy environment in order to maximize full growth potential via the internalization of the asymmetric knowledge diffusion network between regions.

While prior work emphasizes the importance of agglomeration, knowledge diffusion, and R&D subsidies, these strands are typically studied in isolation. Existing endogenous growth models abstract from spatial heterogeneity, while spatial models rarely embed a microfounded knowledge diffusion network. Moreover, the policy debate has largely focused on either uniform R&D subsidies or local development programs without considering how the network structure of idea flows should shape optimal place-based policies. My paper fills this gap by (i) estimating a state-level knowledge diffusion network from patent citation lags, (ii) embedding it in a dynamic growth model with mobile inventors and firms, and (iii) using the framework to design optimal spatial R&D subsidies.

The rest of the paper is organized as follows. Section 2 outlines the data source used in this study and provides empirical regularities in the spatial allocation of workers and inventors as well as heterogeneous citation lags across state pairs in the US. Section 3 introduces the theoretical model and resulting equilibrium along with planner's problem, and model parameters are estimated in Section 4. Section 5 discusses the results for optimal place-based R&D policy in the US. Finally, Section 6 concludes. Proofs of propositions are given in appendix Section A.

2 Empirical regularities

In this section, I focus on two strong phenomena that I observe from the patent data. Firstly, I show that innovation and patenting, and relatedly inventor locations are clustered geographically in the United States, more than other types of economic activity such as employment, population, and GDP. Then, I proceed with the distribution of inventors per employment in locations to understand the extent of the concentration. Some states are asymmetrically populated by inventors relative to overall employment, suggesting heterogeneous demand for researchers across US states. Finally, I analyze geographical concentration in patent citations. I document huge variation in patent citation lags between state-pairs as an evidence for varying degrees of knowledge linkages across US states.

Data. The period I study in this paper is the decade between 2000 and 2010. The geographical unit of analysis is chosen to be all the states in the US including DC (51 states). Worker and inventor allocations over the cross section of states are measured for the median year of the analysis, 2005. For patent citations data, I restrict citing patents between 2000-2010, and cited patents between 1990-2010 (ten more years before 2000). The reason I restrict the sample at 2010 is to remove truncation bias in

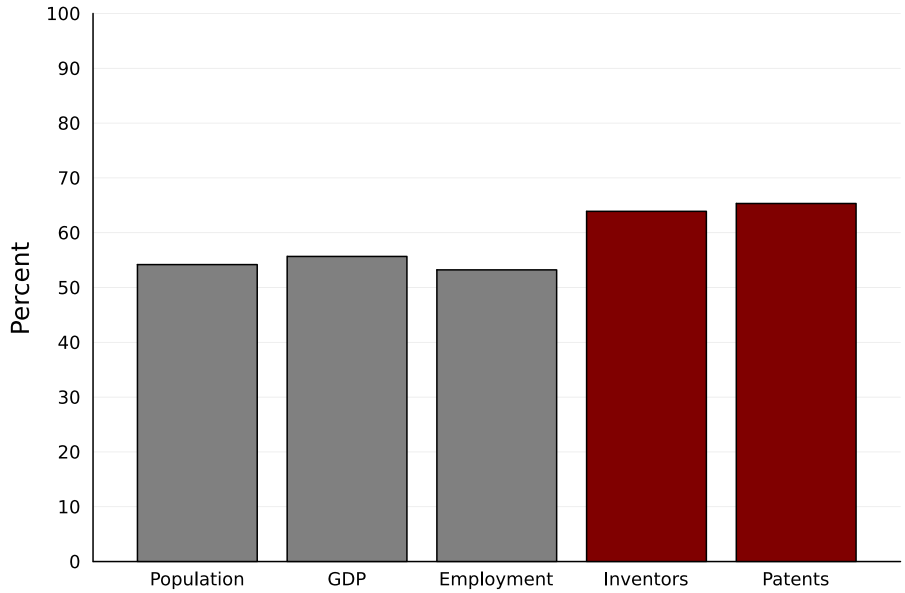


Figure 1: Share of top 10 states with respect to type of economic activity
Note: The set of top 10 states changes among five activities.

patent citations as suggested by Hall et al. (2001). The other reason not to include cited patents that are issued before 1990 is to ensure computational feasibility in my estimation procedure.

State level employment data comes from Census Bureau's Business Dynamics Statistics (BDS). In BDS data, inventors and researchers (in general any type of employment associated with R&D and innovation) are also counted under total employment figures. As the number of total researchers in the US is relatively very small compared to total employment, I do not subtract number of inventors from state level total employment numbers.[5]

For patent citations and spatial allocation of inventors, I use PatentView's disambiguated patent dataset. This dataset is a good fit for the purpose of this study as inventor names and addresses are disambiguated, and hence, allocation of inventors across US states is measurable. The dataset consists of the universe of patents that are applied to USPTO, and it covers the years starting from 1976 until recent years. As explained above, the time range for patent citations is restricted to 2000-2010 for citing patents, and 1990-2010 for cited patents. Hence the maximum citation lag in my data is 20 years with the shortest being zero—citing and cited patents belong to the same year. To determine the location of patents, I use inventor addresses assuming that the R&D is performed in the same location where inventor lives, in line with the model as will be explained in Section 3. Finally, to determine the spatial allocation of inventors across US states, I count the number of unique inventors in 2005, similar to employment.

It should be noted that in the patent data, only inventors that apply for a patent in a year are observed. Therefore, interpreting the number of inventors in the patent dataset as a direct correspondence of the total number of inventors/researchers in states, including ones that do not

[5]This is more of a practical approach as it simplifies the model inversion procedure by removing one iterative step.

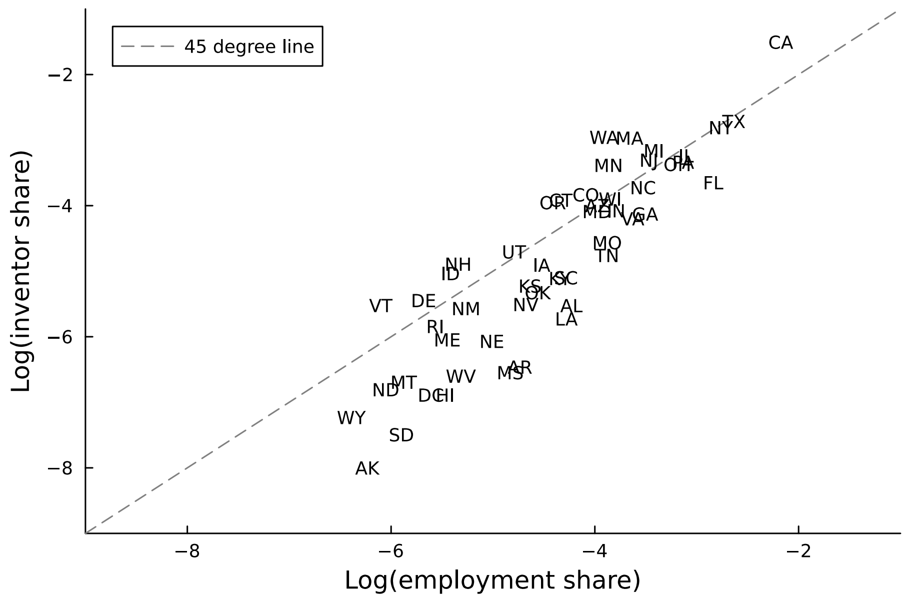


Figure 2: Employment and inventor shares of US states, 2005

Note: Axes are in log scale for visibility. Worker share of a state is calculated as the ratio of total employment of the state divided by the total employment in the US. Similarly, inventor share of a state equals to the ratio of number of inventors located in the state to the total number of inventors.

apply for a patent, is biased, and it is corrected through the lens of the model when I estimate the model parameters, as the model results in several predictions on patenting probability of individual inventors. Full details are described in Section 4.

2.1 Spatial allocation of inventors and employment in the US

Innovative activity that is measured by inventors and patenting is spatially more concentrated than other types of economic activities such as population, GDP, and employment. Figure 1 shows the higher concentration in inventor and patenting allocation across US states as measured by the share of top 10 states.

Figure 2 shows the scatter plot of inventor and employment shares of all US states. California is home to the highest share of employment and inventors. There is strong positive correlation between states' inventor and employment shares. This figure also shows that inventor share of some states such as California (CA), Washington (WA) and Massachusetts (MA) is higher than their employment share. On the contrary, small states such as Alaska (AK), Wyoming (WY), and South Dakota (SD) are populated mainly by production workers rather than inventors.

Figure 3 visualizes the variation in inventor-to-employment ratio on a US map. Inventors prefer some clusters of states more than workers. For example, in the West coast, close states such as Washington, Oregon, California and Idaho are populated by inventors more than workers. Washington is the highest state in terms of inventor to worker ratio with more than three inventors per thousand employment. In the Midwest, Minnesota and Michigan stand out as the states that are preferred more

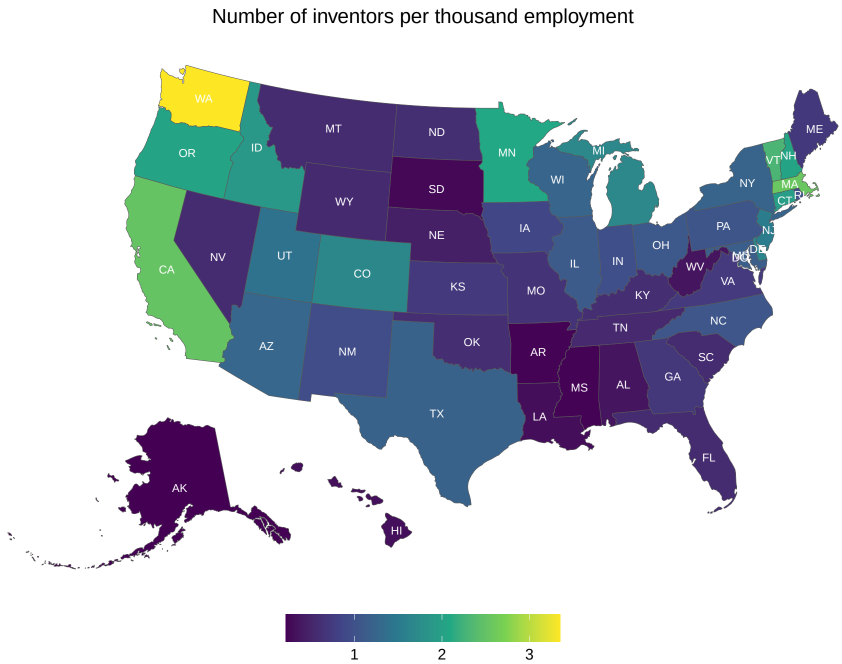


Figure 3: Number of inventors per thousand employment, 2005

by inventors. In the East cost, on the other hand, Massachusetts, Connecticut, Vermont and New Hampshire form a cluster where inventors are relocated relatively more than workers.

States to which inventors are relocated more intensely overlaps partially with the set of states that produce most of the patents in the US in year 2005. These states account for 65% of patents, and 64% of all inventors. For instance, Washington, California, Minnesota, Michigan, and Massachusetts from above are also in the list of top 10 patenting states. Other states in the top 10 list are Texas, New Jersey, New York, Illinois and Pennsylvania. However, inventors do not relocate towards these states disproportionately. This observation suggests that a state's share of inventors alone might not be very informative about its research productivity and the extent of innovative activity, as there are other reasons that could explain the high number of inventors in a state such as amenities, which would also affect the migration choice of production workers in a similar way. Therefore, the variation in inventor-to-worker ratio across locations is a better candidate to identify the specific factors that applies only to inventors when they decide where to locate.

The line of reasoning above is an articulation of a simple supply-demand analysis. Supply of inventors to a region increases with the wage rate offered there. Amenities on the other hand can be considered as a supply shifter. The demand for inventors in a region decreases in inventor wages, while research productivity of the state can be considered as a demand shifter. Higher number of inventors in a state can be the equilibrium outcome of both supply and demand factors. High amenity states would attract more inventors all else equal. However, amenities would also affect the migration decision of workers. Therefore, after controlling for state's employment share, the remaining variation

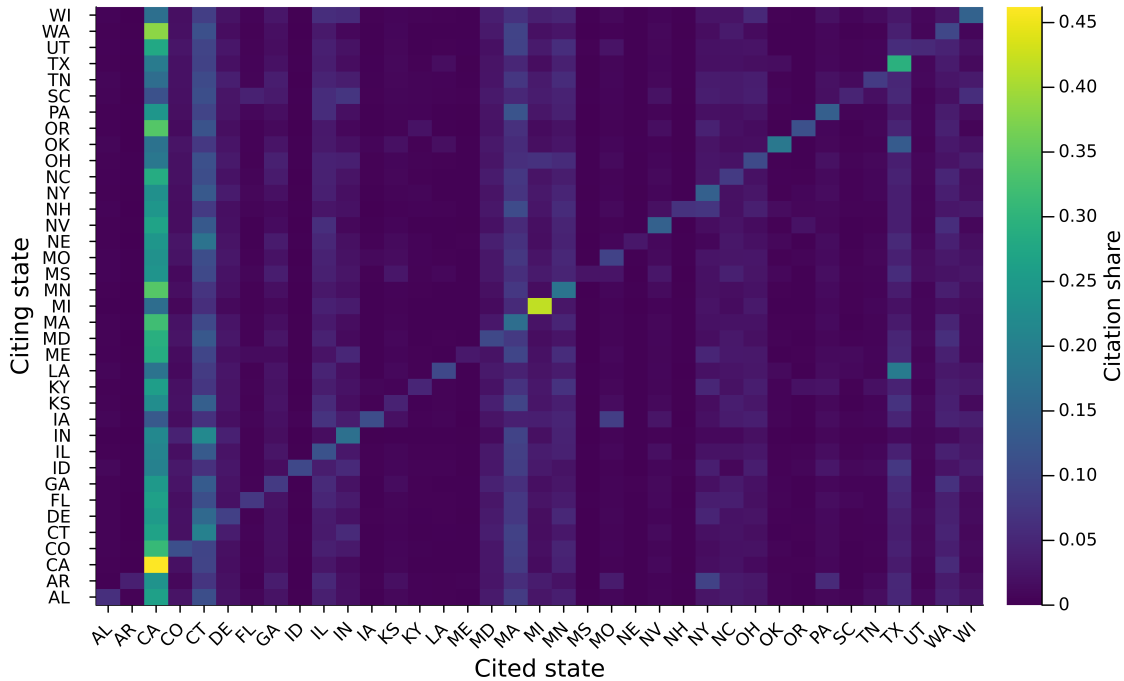


Figure 4: Patent citation shares across US states

in observed inventor shares stems from demand related factors such as research productivity of the location.

2.2 Patent citations across US states

In the sample patent dataset, there are 1,870,743 unique patents that applied for grant between the years 1990 and 2010. Among these, 1,029,211 of them are applied after 2000. The number of observed citations from patents between 2000-2010 to patents from 1990-2010 is 15,727,544.

Figure 4 plots the heatmap of citation shares of citing-cited state pairs between years 2001 and 2010. In particular, the columns correspond to cited states, while citing states are represented by rows. A cell that corresponds to a citing state A and a cited state B shows the share of citations received by patents from B in total citations given by A's patents. As inferred from the citation share matrix, within citation rates are usually higher than out-of-state citation shares. That is, patents are more likely to cite other patents that originate from their states. Furthermore, some cited states such as California (CA), Connecticut (CT), Massachusetts (MA), Texas (TX) stand out as the most cited states by others. It is not a coincidence that these states capture a higher fraction of citations from other states, as they are also the states that produce most of the patents in the US. The citation share of small states is expected to be lower, as the number of patents available for citation in these states is limited. Therefore, Figure 4 is not very informative about the flow of knowledge across states when taken at its face value. However, it still shows, just like patenting activity in the US, patent citations are also

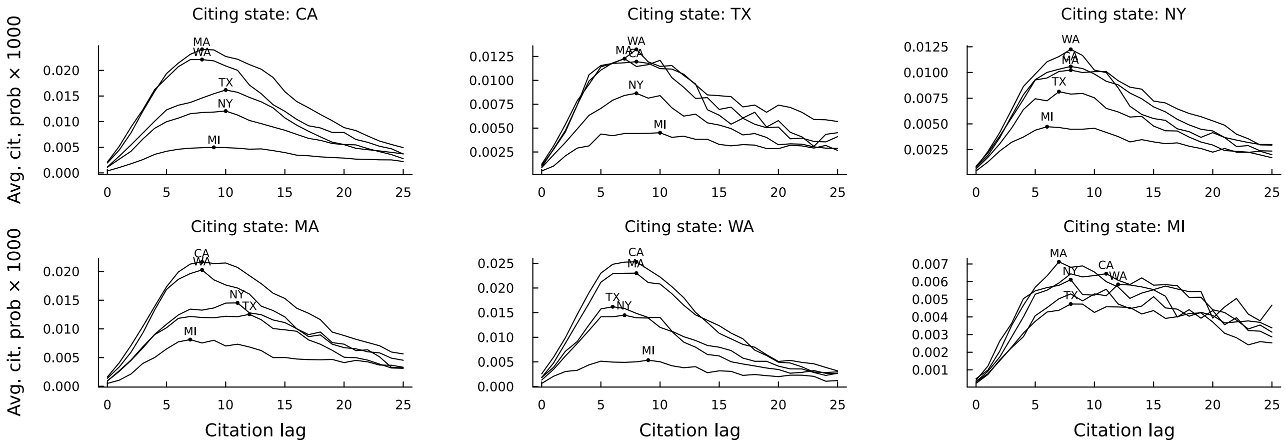


Figure 5: Citation probability of top 6 states as a function of time lag

Note: Within state citations are excluded from figures. For example, in the top left figure, the line corresponding to CA is excluded. Similarly, in the middle top figure, the line corresponding to TX is excluded. The reason is that within-state citation probabilities are much higher than others in levels shadowing the variation across states.

concentrated towards certain states.

In this study, I propose another measure for idea flows between locations from the patent citation data—heterogeneous citation lags between states measured as the time lag between citing and cited patents. This moment is more informative about the pace at which ideas flow across states. Figure 5 shows an example in this regard. For visual clarity, I pick the top 6 states in terms of patenting; California (CA), Washington (WA), Texas (TX), New York (NY), Michigan (MI). Each of six plots represent a citing state. Lines correspond to cited states excluding the citing state itself in order to focus on citation lags between locations. The y-axis plots the average citation probability (multiplied by 1000) calculated as the number of citations divided by the product of the number of patents in citing and cited states (this product gives the number of all possible bilateral connections). The average is taken across citing and cited years for a particular time lag. For example, the citation probability for the lag 5 (years) is the average of citation probabilities that are observed between any two years that has a lag of 5 years such as citing patents are issued in 2001 and cited patents are issued in 1996, citing patents are issued in 2002 and cited patents are issued in 1997, and so on. Thus, Figure 5 illustrates citation probability between any two states as a function of time lag between the time of citation and the time of creation of cited idea.

There are three general messages of Figure 5. The first one is that citation probability is very close to zero when the time lag between citing and cited patents is very short. In other words, the most recent patents receive very little citations. The second message is that citation probability declines as the time lag between citing and cited patents become very large. In other words, old patents are cited less frequently. Finally, citation probability peaks at moderate lags. In other words, certain amount of time is needed for cited patents to be known to others in order to start receiving citations.

Figure 5 reveals another interesting heterogeneity between citing-cited state pairs, i.e. the time lag at which citation probability hits the peak varies across states. For instance, let's focus on the top left figure in which the citing state is California. The probability that patents from Massachusetts

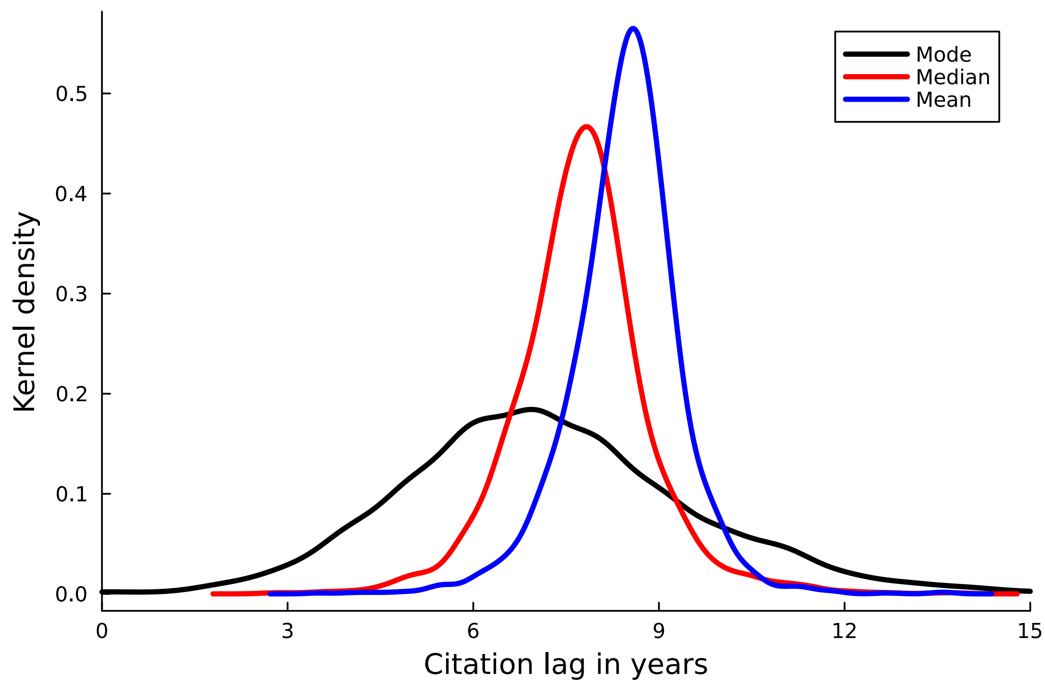


Figure 6: Kernel density estimates of mode, median, and mean citation lag distributions over citing-cited state pairs

Note: Each line corresponds to the distribution of a particular moment of citation lags across citing-cited state pairs. There are $51 \times 51 = 2601$ state pairs in total. For a fixed citing-cited state pair, I calculate the mode citation lag taking total citation counts at each lag being the frequency of lag observations. Then the kernel density estimate of the distribution of mode citation lag across all state pairs is plotted. The same procedure is followed for median and mean citation lag distributions.

(MA) gets citations from patents in California (CA) peaks around a time lag of 7-8 years, while this probability peaks around 10 years for cited Texas (TX) patents. It suggests, on average, CA patents cite MA's patents earlier than TX's patents. Similarly, top middle figure, in which the citing state is Texas (TX), suggests that TX's patents cite Massachusetts's patents (MA) earlier than Michigan's patents (MI). Peak citation lags are not symmetric across citing-cited state pairs. An example is given by the bottom left figure in which the citing state is MA. Although TX patents cite MA patents relatively quicker, MA patents do not cite TX patents that faster. The peak citation lag for TX-MA (citing-cited) pair is around 7 years, while MA-TX peak lag is around 12 years.

The variation in citation lags between patents based on their locations identifies the idea diffusion rates across states. As suggested by the previous discussion, the mode of citation lag between any state-pair is a particular informative moment. In addition to mode citation lag, one can also consider median and mean lags being other moments that identify the average time lag in diffusion of ideas between locations. There is enough variation between state-pairs in terms of all these moments. Figure 6 shows the kernel density estimates of mode, median, and mean citation lag distributions over state-pairs, revealing the extent of heterogeneity in citation lag moments. On average, mode citation lag is shorter than median and mean citation lags suggesting a right-skewed distribution of citation lags for any given citing-cited state pair.

As will be described in Sections 3 and 4, the knowledge spillover network between states is parameterized by a matrix of idea diffusion rates. Denoting by j the state where the idea originates, and by i the state where the idea diffuses to, ω_{ij} will be defined as the (idea) diffusion rate parameter from j to i. It is assumed that new ideas diffuse from j to i with a random time lag which is distributed as Exponential(ω_{ij}). Thus, the average time lag in idea diffusion is given by $1/\omega_{ij}$. If ω_{ij} is higher, then ideas diffuse relatively faster from j to i helping inventors in i to benefit the past ideas of j relatively earlier. It is also assumed that ideas become obsolete with an exogenous rate of δ. As details are delegated to Section 4.1, the probability by which an idea originated in j will be visible in i after τ years is given by $e^{-\delta\tau}\left[1 - e^{-\omega_{ij}\tau}\right]$. Assuming that ideas are cited at the time of their diffusion (conditional on not being obsolete by that time), the unknown parameter ω_{ij} will be identified from the relationship between citation probabilities and citation lags for the state pair $i \times j$. As ω_{ij} increases, inventors in i cites patents from j in earlier lags. This is the source of variation used to identify the knowledge spillover network in the US economy.

3 Model

Motivated by the empirical observations, I build an endogenous growth model with migration choices of two types of agents; workers and inventors. I keep the model as simple as possible by modifying important aspects. Spatial aspect of the model arises due to the fact that both workers and inventors are mobile across locations. Firms are identified by their location choices for their single R&D lab in which they hire inventors and generate new ideas for production. Simple structure of the model implies that firms are indifferent where to produce their products, however, they maximize their value—discounted sum of future profits—by choosing the location for their R&D lab at the time of entry.

3.1 Locations

There are N locations denoted by $i, j = 1, \ldots, N$. Locations are heterogeneous ex-ante only in two respects. Firstly, amenities provided by the location are heterogeneous and time invariant. These amenities can be considered as environmental factors, crime rates, public goods in a location which are valued identically by all individuals, and are denoted by A_i. Secondly, locations differ from each other in terms of the resources they provide specifically to the inventors. These factors are, but not limited to, universities, institutions, culture, and more importantly the past ideas that have diffused to the location from the rest, helping inventors build on when they research for new innovative ideas. I denote (endogenous) research productivity of location i by $\alpha_i(t)$. This aspect of locations might be time varying as it is assumed that it is a combination of two factors. The first factor is called fundamental (exogenous) research productivity of the location, and denoted by $\bar{\alpha}_i$. The second factor is the volume of past innovative ideas that have accumulated in location i up to time t, and is denoted by $K_i(t)$. Researchers in location i are assumed to build on these ideas when they come up with new ideas. This respect of the model reflects the idea of intertemporal spillovers in idea generation process, which is a prominent feature of endogenous growth models, by supplementing it with a spatial aspect. More details on idea diffusion and accumulation process are described in Section 3.6.

I assume a specific functional form for the formation of $\alpha_i(t)$ from these factors

$$\alpha_i(t) = \bar{\alpha}_i^{1-\varphi} K_i(t)^{\varphi}, \qquad \varphi \in [0,1] \tag{3.1}$$

That is, endogenous research productivity is concave in the number of ideas diffused, $K_i(t)$.

3.2 Preferences

Total population consists of two types of individuals, workers and researchers. The aggregate measure of workers in the model, denoted by $\bar{L}$, is normalized to one. Similarly, aggregate measure of inventors is also a constant parameter denoted by $\bar{R}$. Both types of individuals are mobile across locations, earn labor income, and consume hand-to-mouth every period. Specifically, they have no access to financial markets. Workers, denoted by superscript $T = L$, work for the final good production in locations. On the other hand, researchers, denoted by superscript $T = R$, work for intermediate good firms in order to perform research and innovation. Wage rate of type T in location i is denoted by $W_i^T(t)$. Both types of agents discount future utility with a rate of $\rho > 0$.

Per period utility flow rate is denoted by $U_i^T(t)$ and equals to

$$U_i^T(t) = A_i \varepsilon_i C_i^T(t) \tag{3.2}$$

where A_i is location specific, time invariant amenity of location i, $C_i^T(t)$ is the rate of consumption of type T agent in location i. Finally, ε_i denotes individual's idiosyncratic taste for the location. Budget constraint is given by

$$C_i^T(t) = \left[1 + d(t) - \tau(t)\right] W_i^T(t) \tag{3.3}$$

For simplicity, I assume that firms are owned by a national portfolio, and profits are rebated to individuals proportional to their labor income. Therefore, $d(t) \times W_i^T(t)$ is the amount of dividends distributed to an agent with a labor income of $W_i^T(t)$. Labor income tax rate is denoted by $\tau(t)$. Tax rate is not location specific, but it can be potentially time varying, as the tax revenue collected is only used to finance place-based R&D subsidies, as explained later. Government adjusts the tax rate every period to finance its total R&D subsidy expenditures.

3.3 Migration

Agents update their location preferences, $\epsilon \equiv \{\varepsilon_i\}_{i=1}^N$, with a Poisson arrival rate of $\zeta > 0$. Arrival of this shock is independent across individuals, time, and locations. Location taste for each i is independently drawn from a Fréchet distribution

$$\varepsilon_i \overset{\text{ind}}{\sim} \text{Fréchet}\,(\xi, 1)$$

whose shape parameter is given by $\xi > 1$. Lower ξ indicates greater heterogeneity in location taste across individuals. After receiving new location tastes, including for the location they currently live in, individuals decide whether to move to another location or not. Migration between any pair of locations is costless.

Let $\mathcal{U}_i^T(\varepsilon, t)$ denote the discounted life-time utility, simply value, of type T living in location i with a taste of ε. Agents choose to migrate to the location that provides the highest value for themselves. That is, ex-post, they solve the following maximization problem after the realization of a location taste vector $\mathbf{e} \equiv \{e_i\}_{i=1}^N$ where e_j denotes the j$^{\text{th}}$ component of vector $\mathbf{e}$

$$\max_{j=1,\ldots,N} \mathcal{U}_j^T(e_j, t) \tag{3.4}$$

Ex-ante, location tastes are uncertain for agents. It is useful to define the following function which can be defined as the expected value of arrival of taste shocks

$$\bar{\mathcal{U}}^T(t) \equiv \int \left\{ \max_{j=1,\ldots,N} \mathcal{U}_j^T(e_j, t) \right\} f_\epsilon(\mathbf{e})\, d\mathbf{e} \tag{3.5}$$

where $f_\epsilon(\cdot)$ denotes the joint density of location taste vector.[6] The idea behind (3.5) is that agents decide to relocate to the maximum value location. Equipped with this notation, we can derive the HJB equation for the evolution of $\mathcal{U}_i^T(\varepsilon, t)$ as follows

$$\rho \mathcal{U}_i^T(\varepsilon, t) = A_i \varepsilon C_i^T(t) + \zeta \left[\bar{\mathcal{U}}^T(t) - \mathcal{U}_i^T(\varepsilon, t) \right] + \frac{\partial \mathcal{U}_i^T(\varepsilon, t)}{\partial t} \tag{3.6}$$

Derivation of (3.6) from discrete time can be found in Appendix A.1. In words, agents derive a flow rate of utility of $A_i \varepsilon C_i^T(t)$ in location i. With a rate of ζ, they draw new tastes for locations, and decide to migrate after which they earn an expected life-time utility of $\bar{\mathcal{U}}^T(t)$. Thus the utility return from drawing new taste shocks is given by $\bar{\mathcal{U}}^T(t) - \mathcal{U}_i^T(\varepsilon, t)$. The last term stands for the time appreciation in the value function.

Finally, the equilibrium number of workers and inventors located in i are denoted by $L_i(t)$ and $R_i(t)$, respectively, satisfying that $\sum_i L_i(t) = \bar{L}$ and $\sum_i R_i(t) = \bar{R}$.

3.4 Production

There are two types of goods produced in the economy. Final good is used for consumption and as an input for intermediate good production. The price of final good is normalized to one. Intermediate goods are used as an input for the production of the final good. Workers are employed in final good sector in each location. All goods are shipped across locations without any costs. Thus, the place of production is not important apart from the fact that they create demand for labor.

[6] The function $\bar{\mathcal{U}}^T(t)$ does not possess a location subscript as $\mathcal{U}_j^T(e_j, t)$ is maximized over the whole set of locations. Intuitively, it is because of the assumption that migration is costless. That is, the migration decision does not depend on the origin location, as moving between any location pair has zero cost.

Final goods. Production function of final good in location i is given by

$$Y_i(t) = \bar{A} L_i(t)^{\beta} \left[\exp\left(\int_0^1 \log k_i(\nu, t) d\nu \right) \right]^{1-\beta} \tag{3.7}$$

All locations have access to the identical production technology (3.7). $\bar{A}$ is a constant term to simplify algebra.[7] $L_i(t)$ denotes the amount of workers employed in location i, and $k_i(\nu, t)$ denotes the amount of intermediate good $\nu \in [0, 1]$ demanded by location i. The elasticity of substitution between factors of production is one, and β represents the share of labor in the production of final good.

Final goods are produced in perfectly competitive markets. As trade is costless, the price of intermediate good ν is identical in all markets. Denoting this price by $p(\nu, t)$, demand functions for production factors are given by

$$W_i^L(t) L_i(t) = \beta Y_i(t)$$
$$p(\nu, t) k_i(\nu, t) = (1 - \beta) Y_i(t)$$

Intermediate goods and R&D. A unit measure of intermediate goods are differentiated, and each variety is denoted by $\nu \in [0, 1]$. These varieties are simply called products throughout the paper. Each product is produced by a single intermediate good firm (or simply firm) in equilibrium, however, a firm can produce multiple products. The intermediate good sector is identical to Klette and Kortum (2004)'s model of heterogeneous firms and innovation. In addition to the fact that firms are defined by the portfolio of products they produce, as in Klette and Kortum (2004), I make only one addition to their structure, i.e. firms establish their single R&D lab in a location, and they perform their R&D and innovation in this lab by hiring researchers. I assume that firms cannot change the location of the R&D lab throughout their life-time.

Intermediate goods are produced from final goods with a linear technology

$$k(\nu, t) = a(\nu, t) y(\nu, t)$$

where $a(\nu, t)$ is productivity, and $y(\nu, t)$ is the final good used in production. Unit elastic demand for intermediate goods implies that the firm that holds the most frontier technology for a good ν captures the whole market, and charges a markup so that the price charged is just equal to the marginal cost of the second most productive firm. As the price of the final good is normalized to one, the price of variety ν equals $p(\nu, t) = \lambda a(\nu, t)^{-1}$ where $\lambda > 1$ denotes the innovation step size as described below.

Firms invest in R&D to expand their product portfolio. Upon a successful innovation, the firm comes up with a new production technology for a random product $\nu' \in [0, 1]$ drawn from a Uniform distribution on the unit interval, improving previous productivity $a(\nu', t)$ by a constant factor of $\lambda > 1$, and obtains the monopoly of the good with a higher productivity level of $\lambda a(\nu', t)$.[8] As the price of

[7] Given the innovation step size parameter $\lambda > 1$ defined below, $\bar{A} = \left(\frac{\lambda}{1-\beta}\right)^{1-\beta}$.

[8] As all firms have a countable number of products in their portfolio, the probability of the event in which the firm

final good is one in all locations, the relative marginal cost of past incumbent to that of the current incumbent in any product line is always equal to λ. This is the reason as to why the maximum markup that can be charged equals to λ as described above. The flow of profits per unit of time is then equals to $\pi Y(t)$ for all products $\nu \in [0,1]$, where $\pi \equiv \left(1-\lambda^{-1}\right)(1-\beta)$ and $Y(t) \equiv \sum_n Y_n(t)$ is the the total output produced in the economy. As the rate of profit per product does not depend on the location of production, firms are indifferent in where to produce their products. The production locations of intermediate goods are indeterminate in the model because of the absence of trade costs, and the identical cost of production factor for all firms, i.e. the final good, no matter where they locate their production plant.

Let n denote the number of products owned by a firm, and i denote the location of the R&D lab of the firm. The R&D production technology of the firm is given by

$$Z_i(n,t) = [\alpha_i(t) R_i(n,t)]^{\frac{1}{\theta}} n^{1-\frac{1}{\theta}}, \qquad \theta > 1 \tag{3.8}$$

In equation (3.8), $Z_i(n,t)$ denotes the rate of innovation which is the Poisson arrival rate of a successful innovation, and R denotes the number of researchers employed in the R&D lab located in i. Researchers in location i benefit from the location research productivity $\alpha_i(t)$ as described in Section 3.1. Moreover, it is assumed that larger firms in terms of the number of products owned are more productive in R&D. The parameter θ governs the curvature of the innovation production function with respect to the number of inventors employed.

Firms are owned by all individuals in the economy. As they have linear preferences over time, the rate of interest equals to the time discount rate ρ. Thus, the HJB function for the value function of a firm with n products and located in i can be written as follows

$$\begin{aligned}\rho \mathcal{V}_i(n,t) - \dot{\mathcal{V}}_i(n,t) = & \underbrace{n\pi Y(t)}_{\text{Profit}} + \underbrace{nx(t)\left[\mathcal{V}_i(n-1,t) - \mathcal{V}_i(t)\right]}_{\text{Value loss from creative destruction}} \\ & + \max_R \Big\{ \underbrace{-(1-s_i)\, W_i^R(t) R}_{\text{Cost of R\&D}} + \underbrace{\left[\alpha_i(t) R\right]^{\frac{1}{\theta}} n^{1-\frac{1}{\theta}}}_{\text{Innovation rate } Z_i(n,t)} \underbrace{\left[\mathcal{V}_i(n+1,t) - \mathcal{V}_i(n,t)\right]}_{\text{Value gain from inn.}} \Big\}\end{aligned} \tag{3.9}$$

where $\mathcal{V}_i(n,t)$ denotes the sum of discounted future profits. The right hand side of (3.9) has multiple terms. The first one is the total rate of profits generated from the whole product portfolio. The second term is the loss in firm value due to creative destruction. $x(t)$ denotes equilibrium aggregate rate of creative destruction per product, which is defined as the total rate of innovation per unit of time from all locations in the economy. Firms take this as given when deciding how much to invest in R&D. As the firm has n products, the total rate of creative destruction that the firm faces equals to $nx(t)$. When other firms obtain a superior technology, the incumbent firm loses one of the products from its portfolio. If the firm had a single product, then the firm exits in such a case. The last term is the value stemming from the R&D investments. The first term in the maximization problem is the total cost of innovation which equals to the wage bill of researchers employed, after s_i portion of it is subsidized by

invents on one of its products equals zero.

the government. In particular, $W_i^R(t)$ denotes the equilibrium wage rate of inventors in location i, and $s_i \in [0,1]$ denotes the R&D subsidy rate in the location. The second term indicates the expected return from innovation which is the product of the rate of innovation and the value gain after firm adds an additional product to its portfolio.

Proposition 1 *The solution to the HJB equation* (3.9) *has the form* $\mathcal{V}_i(n,t) = nv_i(t)Y(t)$ *where* $v_i(t)$ *denotes the normalized monopoly value of owning a product. Furthermore, per product value* $v_i(t)$*, per product innovation rate, defined as* $z_i(t) \equiv \frac{Z_i(n,t)}{n}$*, and per product inventor employment, defined as* $r_i(t) \equiv \frac{R_i(n,t)}{n}$ *are independent of the size of the firm's portfolio n and satisfy the following set of equations*

$$z_i(t) = \left[\alpha_i(t) r_i(t)\right]^{\frac{1}{\theta}} \tag{3.10}$$

$$(1-s_i)\, w_i^R(t) r_i(t) = \frac{1}{\theta}\left[\alpha_i(t) r_i(t)\right]^{\frac{1}{\theta}} v_i(t) \tag{3.11}$$

$$\dot{v}_i(t) = \left[\rho - g(t) + x(t) - \frac{\theta-1}{\theta} z_i(t)\right] v_i(t) - \pi \tag{3.12}$$

where $w_i^R(t) \equiv \frac{W_i^R(t)}{Y(t)}$ *denotes inventor wage in i normalized by aggregate output, and* $g(t) \equiv \frac{\dot{Y}(t)}{Y(t)}$ *is the growth rate of aggregate output.*

Proof. See Appendix A.2. ■

Proposition 1 states that the value of a product is independent of the size of the firm which is measured by n. This results stems from the Cobb-Douglass specification for the innovation production function (3.8).[9] Furthermore, per product line innovation rates and inventor employments are also independent of firm size. Equation (3.11) gives the demand for inventors by incumbent firms from a location. Ceteris paribus, high research productivity in a location causes higher demand for inventors for any given wage level. Therefore, $\alpha_i(t)$ can be regarded as a demand shifter for inventors across locations.

3.5 Entry

Location of R&D lab of incumbent firms is determined at the time of entry. There is a unit mass of potential entrants in the economy who are frictionlessly mobile across regions. Similar to incumbents, entrants employ researchers to generate a superior technology on a random product line $\nu \in [0,1]$. A potential entrant in location i who employs $\tilde{r}_i(t)$ inventors generates an innovation rate of $\tilde{z}_i(t)$ which is given by

$$\tilde{z}_i(t) = \frac{1}{f}\left[\alpha_i(t)\tilde{r}_i(t)\right]^{\frac{1}{\theta}} \tag{3.13}$$

The parameter f represents entry costs that are common to all locations, and the curvature parameter θ is same across entrants and incumbents. Importantly, inventors benefit from location specific research productivity $\alpha_i(t)$ whether they are employed by incumbent firms or entrants. Denoting the value of

[9] A more detailed discussion on the implications of this R&D function can be found in Akcigit and Kerr (2018).

entry in location i by $\tilde{\mathcal{V}}_i(t)$, each potential entrant located in i solves the following entry problem

$$\tilde{\mathcal{V}}_i(t) \equiv \max_{\tilde{r}} \Big\{ \underbrace{-(1-s_i)\, W_i^R(t)\tilde{r}}_{\text{Cost of R\&D}} \;+\; \underbrace{\frac{1}{f}\,[\alpha_i(t)\tilde{r}]^{\frac{1}{\theta}}}_{\text{Inn. rate } \tilde{z}_i(t)} \underbrace{\mathcal{V}_i(1,t)}_{\text{Return}} \Big\} \tag{3.14}$$

The return from innovation for entrants is equal to the market value of an incumbent firm on the same location that starts with a single product.

Proposition 2 *Let normalized value of being an entrant in location i be defined as* $\tilde{v}_i(t) \equiv \frac{\tilde{\mathcal{V}}_i(t)}{Y(t)}$. *Then,*

$$\tilde{v}_i(t) = \frac{\theta-1}{\theta}\tilde{z}_i(t)v_i(t) \tag{3.15}$$

Furthermore, inventor employment of a potential entrant is proportional to that of incumbent firms in their location. As a result, per potential entrant innovation rate is also proportional to per product innovation rate of the location. That is,

$$\tilde{r}_i(t) = \frac{1}{F}r_i(t) \tag{3.16}$$

$$\tilde{z}_i(t) = \frac{1}{F}z_i(t) \tag{3.17}$$

where F is a composite parameter defined as $F = f^{\frac{\theta}{\theta-1}}$.

Proof. See Appendix A.3. ■

Proposition 2 can be proven easily by combining first order conditions to incumbent and entrant problems, (3.9) and (3.14). The implication of this proposition is that entrant choices are closely linked to incumbent firms in their location. The reason is that they have access to a similar R&D production function with incumbent firms, and inventors benefit the location specific R&D resources, $\alpha_i(t)$, both in incumbent and entrant firms in a location. This structure is particularly chosen so that entry part of the model simplifies considerably. The only mission of entrants in the model is to give rise to new firms that exit frequently due to creative destruction $x(t)$. However, entrants' location choice is not trivial. Indeed, they are indifferent in equilibrium between locations to perform R&D and enter to the market. Labeling this equilibrium condition as free entry condition across locations, it can be formally stated as[10]

$$\tilde{v}_i(t) = \tilde{v}_j(t), \qquad \forall i, j, t \tag{3.18}$$

The free entry condition (3.18) pins down the equilibrium mass of potential entrants across locations which are denoted by $\{\tilde{\psi}_i(t)\}_i$ such that $\sum_i \tilde{\psi}_i(t) = 1$ for all t. Similarly, the total measure of product lines owned by firms from location i in equilibrium is denoted by $\psi_i(t)$ such that $\sum_i \psi_i(t) = 1$. The variable $\psi_i(t)$ has an endogenous evolution over time as a result of firm innovation choices and entry rates in the location. Intuitively, it increases in the number of potential entrants in i, $\tilde{\psi}_i(t)$ as more

[10]Normally, this condition should be stated at the non-normalized levels of entry values, i.e. $\tilde{\mathcal{V}}_i(t) = \tilde{\mathcal{V}}_j(t)\ \forall i, j, t$.

entry means higher survival rate of location i firms compared to other regions. Formal derivations are delegated to Appendix A.4.

3.6 Knowledge diffusion across locations

As described in Section 3.1, research productivity in locations depends on endogenous flow of past ideas within the country. I assume that each innovation embeds a measure of ideas normalized to one. After the invention of these ideas in an origin location j, they diffuse to the rest of the economy in order to lay the foundation for the new ideas to be invented, possibly combined with other ideas that have diffused from somewhere else. However, the diffusion process is not homogeneous and perfect across location pairs. I assume that ideas diffuse between locations with a random time lag. Specifically, let $\omega_{ij} > 0$ be called the rate of diffusion from j to i. Then, the time lag for which an idea originated in location j diffuses to location i is a random variable distributed as Exponential (ω_{ij}). The parameter of this distribution, ω_{ij}, varies across origin-destination pairs, and it is possible that $\omega_{ij} \neq \omega_{ji}$. It follows from this structure that the mean time lag of idea diffusion from j to i is equal to $1/\omega_{ij}$. As $\omega_{ij} \to 0$, ideas never diffuse from j to i in finite time. On the contrary, as $\omega_{ij} \to \infty$, the diffusion becomes instantaneous. The $N \times N$ matrix $\Omega = [\omega_{ij}]$ holds the diffusion rate parameters, and it is called the knowledge network throughout the paper.

In line with the empirical evidence in Section 2.2, it is also assumed that ideas get obsolete at an exogenous rate of $\delta > 0$.[11] As ideas get older, they are more likely to be replaced by new and better ideas.

In order to derive the evolution of $K_i(t)$, where i is called the destination location, it is required to define a variable which represents the number of ideas that are invented in j, but have not yet diffused to i by time t. This variable is denoted by $I_{ij}(t)$. Then, the law of motion of $K_i(t)$ can be derived as

$$\dot{K}_i(t) = \sum_{j=1}^{N} \omega_{ij} I_{ij}(t) - \delta K_i(t) \tag{3.19}$$

The first term on the right hand side is a summation across all locations. The flow of ideas to location i from j is equal to the rate of diffusion times the stock of ideas available for diffusion. On the other hand, the second term captures the obsolescence of accumulated ideas.

How does $I_{ij}(t)$ evolves over time? Let $x_j(t)$ be the total rate of innovation in origin location j. Then, it is equal to $x_j(t) = \psi_j(t) z_j(t) + \tilde{\psi}_j(t) \tilde{z}_j(t)$. The inflow to the stock of not-yet-diffused ideas to a particular location i equals the number of ideas embedded in an invention, which is normalized to one, times the rate of innovation, $x_j(t)$. On the other hand, the outflow of ideas from this stock is due to either diffusion to i or obsolescence. Hence, we can show that $\dot{I}_{ij}(t)$ equals to

$$\dot{I}_{ij}(t) = x_j(t) - (\omega_{ij} + \delta)\, I_{ij}(t) \tag{3.20}$$

Equation (3.19) suggests that the rate at which $K_i(t)$ grows over time increases with ω_{ij}, and the size

[11] Obsolescence of old ideas can be endogenized by the creative destruction process of frontier technologies.

of the stock of ideas waiting to be diffused, $I_{ij}(t)$. This stock, on the other hand, is positively correlated with the rate of innovation j, i.e. $x_j(t)$. Therefore, connectedness represented by ω_{ij} is not the only determinant of the size of past ideas available for use in i. Locations that are particularly connected to innovation hubs, i.e. locations with high $x_j(t)$, benefit from knowledge spillovers relatively more.

3.7 Market clearing conditions

Workers. Supply of workers in a location, determined from their migration decision (3.4), is equal to the labor demand from final good producers in the location. Clearing intermediate good markets along with local worker markets in each location gives rise to a very simple solution for the wage rate of workers. Delegating the derivations to Appendix A.5, we can show that worker wage rate is common across locations, and is given by

$$W_i^L(t) = W^L(t) = \frac{\beta}{\bar{L}} Y(t), \qquad \forall i = 1, \ldots, N \tag{3.21}$$

In the absence of trade costs, the marginal productivity of workers across locations are equal, hence they earn equal wages, which is proportional to aggregate output at all times. Thus, when workers move across locations, they only value relative amenities. This implication of the model allows me to control for amenity differences across locations by matching observed worker allocation. Therefore, the remaining variation in inventor-to-worker ratio informs relative inventor wages, which is heterogeneous across locations.

Total output of the economy equals to $Y(t) = \mathcal{A}(t)^{\frac{1-\beta}{\beta}} \bar{L}$, where $\mathcal{A}(t)$ is the aggregate productivity index defined by

$$\mathcal{A}(t) \equiv \exp\left[\int_0^1 \log a(\nu, t) d\nu\right] \tag{3.22}$$

which is a unit elastic aggregation across productivity of all intermediate goods. The source of growth stems from innovations at the intermediate goods level. Furthermore, as shown in Appendix A.6, the growth rate of aggregate productivity equals to $\frac{\dot{\mathcal{A}}(t)}{\mathcal{A}(t)} = \log(\lambda)\, x(t)$. Thus, the growth rate of output is given by

$$g(t) = \frac{1-\beta}{\beta} \log(\lambda)\, x(t) \tag{3.23}$$

Inventors. Total supply of inventors in a location, $R_i(t)$, is determined by inventor migration choice given by (3.4). The demand, on the other hand, is equal to total inventor employment in a location, which is the sum of incumbent's and entrant's demand for inventors. Hence, market clearing condition for inventors in location i can be stated as

$$R_i(t) = \psi_i(t) r_i(t) + \tilde{\psi}_i(t) \tilde{r}_i(t) \tag{3.24}$$

Government budget constraint. Government finances location specific R&D subsidies from the taxation of individuals' labor income. It is assumed that it holds period-by-period

$$\sum_{i=1}^{N} s_i W_i^R(t) R_i(t) = \sum_{i=1}^{N} \tau(t) W_i^L(t) L_i(t) + \sum_{i=1}^{N} \tau(t) W_i^R(t) R_i(t)$$
$$\implies \tau(t) = \frac{\sum_{i=1}^{N} s_i W_i^R(t) R_i(t)}{\sum_{i=1}^{N} W_i^L(t) L_i(t) + W_i^R(t) R_i(t)} \tag{3.25}$$

Total profits and their allocation across agents are derived in Appendix A.7.

3.8 Equilibrium and predictions of the model

We can now proceed with the equilibrium properties and the predictions of the model on equilibrium wage rate of inventors and inventor allocation across locations. The particular equilibrium that is considered in the paper is the balanced growth path (BGP) equilibrium in which the growth rate of the economy $g(t)$ is constant over time. Moreover, in this equilibrium, the growth rate of inventor wages in all locations are equal to the growth rate of output. Thus, the model variables stay stationary in this equilibrium after normalizing them with the aggregate output $Y(t)$. In what follows, I will conjecture that the model admits a BGP equilibrium, derive its properties, and then finally show that the initial conjecture holds.

Knowledge network and research productivity. In BGP, location innovation rates x_j are time invariant. Under this conjecture, the system of differential equations given by (3.19) and (3.20) have a stationary solution given by $K_i = \frac{1}{\delta} \sum_{j=1}^{N} \frac{\omega_{ij}}{\omega_{ij}+\delta} x_j$. Thus location research productivity $\alpha_i = \bar{\alpha}_i^{1-\varphi} K_i^{\varphi}$ is also constant over time. Moreover, the rate of creative destruction which is the total innovation rate in the economy is a constant and equals to $x = \sum_i x_i$.

Innovation rates. Replacing the aggregate creative destruction rate x into (3.23) implies that the growth rate of the economy equals to $g = \frac{1-\beta}{\beta} \log(\lambda)\, x$. The second conjecture of the BGP equilibrium is that the normalized inventor wage w_i^R and inventors per product r_i are constant over time. Under these conjectures, we can show that $\dot{v}_i(t) = 0$, and stationary values z_i, r_i and v_i satisfy the system of equations given by (3.10), (3.11) and (3.12). Importantly, we have

$$v_i = \frac{\pi}{\rho - g + x - \frac{\theta-1}{\theta} z_i} \tag{3.26}$$

Equation (3.15) combined with free entry condition (3.18) implies that per product rate of innovation in locations are equal to a common rate, $z_i = z_j = z$ for all i, j. Using the relationship between incumbent and entrant innovations given by equation (3.17), we also have that entrants in all locations choose the same rate of innovation $\tilde{z}_i = \tilde{z} = z/F$.

Proposition 3 *Let ψ_i denote the total measure of product lines owned by incumbent firms that are located in i, and let $\tilde{\psi}_i$ denote the measure of potential entrants located in i. Then, in BGP equilibrium,*

$$\psi_i = \tilde{\psi}_i = \frac{\alpha_i R_i}{\sum_{j=1}^{N} \alpha_j R_j} \tag{3.27}$$

Proof. See Appendix A.8. ■

Although firms and entrants choose equal rates of innovations in any location, the difference between regions in terms of innovative activity stems from the extensive margin. That is, in equilibrium, firms located in more research productive regions obtain a higher share of market ownership which is measured by the mass of product lines owned by local firms. The intuition is as follows. In equilibrium, more entrants prefer high research productive locations. Therefore, the entry rate in those locations are higher. Since all firms in the economy face the same exit probability which is implied by the aggregate creative destruction rate x, a startup cohort from more productive locations are more successful in surviving in the market because of their large population due to high entry rate. Thus, in equilibrium, firms from research productive locations survive better and capture a larger fraction of product markets in overall economy. Although I do not test the spatial firm dynamics predictions of the model in this paper, the difference across locations stemming from heterogeneous firm dynamics allows me to explain the fundamental source of high demand for inventors in certain locations. In other words, in the model, the reason for high inventor demand in certain locations is not directly due to the presence of high volume of innovative firms there. Instead, there is another factor, endogenous research productivity of locations, which gives rise to both phenomena simultaneously, i.e. high inventor demand and high number of innovative firms.

Inventor wage across locations. Another important prediction of the model is that inventor wages are proportional to research productivity of locations. Next proposition shows this result.

Proposition 4 *Let w_i^R denote the inventor wage rate in equilibrium normalized by aggregate output. Then*

$$w_i^R = \frac{1}{\theta} z^{1-\theta} v \frac{\alpha_i}{1 - s_i} \tag{3.28}$$

where z is per product line innovation rate common to all locations, and $v = \frac{\pi}{\rho - g + x - \frac{\theta-1}{\theta} z}$ following from (3.26), and $\alpha_i = \bar{\alpha}_i^{1-\varphi} K_i^{\varphi}$ is the research productivity of location i.

Proof. See Appendix A.9. ■

Equilibrium inventor wage in a location increases with research productivity of the location and the subsidies provided for R&D activities. This prediction of the model allows me to pin down relative research productivities of locations by exactly matching inventor allocation across US states. Next section describes the migration behavior of agents in BGP, and shows the resulted allocation of workers and inventors across space.

Migration and inventor allocation in space. In order to simplify the migration problem of agents given by (3.4), we first need to solve the value function $\mathcal{U}_i^T(\varepsilon, t)$ in BGP.

Proposition 5 *Agent value function $\mathcal{U}_i^T(\varepsilon, t)$ as defined in Section 3.3 has an analytical solution in BGP equilibrium as follows*

$$\mathcal{U}_i^T(\varepsilon, t) = \frac{A_i \varepsilon C_i^T(t)}{\rho + \zeta - g} + \frac{\zeta}{(\rho + \zeta - g)(\rho - g)} \Gamma\left(1 - \frac{1}{\xi}\right) \left[\sum_{j=1}^{N} \left(A_j C_j^T(t)\right)^{\xi}\right]^{\frac{1}{\xi}} \tag{3.29}$$

where $\Gamma(\cdot)$ is Gamma function, and $C_i^T(t) = (1 + d - \tau) w_i^T Y(t)$ is consumption of type-T in i that is proportional to aggregate output. Thus, $\mathcal{U}_i^T(\varepsilon, t)$ is also proportional to $Y(t)$.[12]

Proof. See Appendix A.10. ■

Having equipped with agent values, the migration choice (3.4) simplifies considerably in BGP as stated by the next proposition.

Proposition 6 *Let $(i^T)^\star$ be the location choice of agents of type-T in BGP, conditional on a set of location tastes given by vector* **e**. *Then,*

$$\left(i^T\right)^\star = \arg\max_j \left\{A_j e_j w_j^T\right\} \tag{3.30}$$

where e_j is the j^{th} component of **e**. *Furthermore, let γ_i^T be the fraction of type-T population located in i. Given worker and inventor wages in* (3.21) *and* (3.28), *the migration choice* (3.30) *implies*

$$\gamma_i^L = \frac{A_i^\xi}{\sum_{j=1}^N A_j^\xi} \tag{3.31}$$

$$\gamma_i^R = \frac{\gamma_i^L \left(\frac{\alpha_i}{1-s_i}\right)^\xi}{\sum_{j=1}^N \gamma_j^L \left(\frac{\alpha_j}{1-s_j}\right)^\xi} \tag{3.32}$$

Thus, number of workers and inventors in locations can be found as $L_i = \gamma_i^L \bar{L}$ and $R_i = \gamma_i^R \bar{R}$.

Proof. See Appendix A.11. ■

Proposition 6 forms the basis for the identification of location specific research productivities. Heterogeneity in amenities across locations, which is an important ingredient in inventor supply to local labor markets, is controlled for by observed worker allocation in space. Simple structure of the model aggregates possibly many different characteristics of locations under a single residual, the amenity A_i. The assumption needed is that any such characteristics affect both types of individuals identically when they decide where to relocate. Further implication of (3.32) is that inventor-to-worker

[12] The analytical expressions of d and τ in BGP equilibrium are given in proof.

ratio in a location increases with its research productivity. That is,

$$\frac{\alpha_i}{\alpha_j} = \frac{1-s_i}{1-s_j}\left[\frac{\left(\gamma_i^R/\gamma_i^L\right)}{\left(\gamma_j^R/\gamma_j^L\right)}\right]^{\frac{1}{\xi}} \tag{3.33}$$

Equation (3.33) identifies relative research productivity of locations given taste dispersion parameter ξ, as the right hand side of the equation is observable in the data.[13] Given worker and inventor allocations from the data, we can solve for relative research productivity. As $\alpha_i = \bar{\alpha}_i^{1-\varphi}K_i^{\varphi}$ holds true, we can further decompose α_i in fundamental research productivity of location, $\bar{\alpha}_i$, and network effects captured by K_i, as explained below.

Verifying initial conjectures. The predictions derived up to this point depend on the initial conjectures that x_i, w_i^R and r_i are constant over time. Proposition 4 proves that w_i^R is indeed constant. From equation (3.8), we can show that $r_i = z^{\theta}/\alpha_i$, which does not vary over time. Following proposition proves that x_i is also a constant in BGP equilibrium.

Proposition 7 *In BGP, the total rate of innovation in a location x_i and per product innovation rate z can be derived as follows*

$$z = \left[\frac{F}{1+F}\sum_{i=1}^{N}\alpha_i R_i\right]^{\frac{1}{\theta}} \tag{3.34}$$

$$x_i = z^{1-\theta}\alpha_i R_i \tag{3.35}$$

Further replacing x_i in $x = \sum_i x_i$ implies that the aggregate rate of creative destruction equals to

$$x = \frac{1+F}{F}z \tag{3.36}$$

Thus, aggregate growth rate of the economy is finally

$$g = \frac{1-\beta}{\beta}\log\left(\lambda\right)\frac{1+F}{F}z \tag{3.37}$$

which is proportional to z.

Proof. See Appendix A.12. ■

Proposition 7 verifies the initial conjecture that x_i are constant over time. Moreover, equation (3.35)

[13] As discussed in Section 4, observed inventor allocation in patent data is not exactly equal to true inventor allocation, as the data only consists of the inventors that applied for a patent in a given period of time. As will be shown in the same Section, the model structure allows us to make a connection from the number of "successful" inventors who applied for a patent to true number of inventors including "unsuccessful" ones.

and definition $\alpha_i = \bar{\alpha}_i^{1-\varphi} K_i^{\varphi}$ result in a nonlinear system of equations in $\{K_i\}_i$ such that

$$K_i = \frac{1}{\delta z^{\theta-1}} \sum_{j=1}^{N} \frac{\omega_{ij}}{\omega_{ij}+\delta} \bar{\alpha}_j^{1-\varphi} K_j^{\varphi} R_j \tag{3.38}$$

Equation (3.38) is endogenous in the sense that knowledge spillovers across locations depend on inventor allocation through their effect on innovation intensity in locations. Ceteris paribus, regions that are more connected to the locations with large inventor populations benefit more from knowledge spillovers. The reason is the direct effect of inventor population on idea creation in origin locations. More inventors create more ideas per unit of time, and these ideas spill to other connected locations faster. Furthermore, fundamental research productivity $\bar{\alpha}_j$ and inventors R_j reinforce this effect, as inventors are more likely to migrate to locations with high $\bar{\alpha}_j$.

3.9 Social welfare function and planner's problem

In this section, social welfare function is derived based on agent value functions found in Proposition 5. It is assumed that the social planner cares about the ex-ante expected value of agents, $\bar{\mathcal{U}}^T(t)$, before they draw idiosyncratic taste shocks and migrate to the location that provide the highest value for themselves, which is given by definition (3.5). The function $\bar{\mathcal{U}}^T(t)$ represents the social welfare of type-T agents because the planner internalizes agents' migration decisions based on their idiosyncratic location preferences, and she knows that they would migrate to the highest value locations. It is a good choice in comparing long run equilibria under different counterfactuals, as it abstracts away from transition periods during which agents relocate across locations between regions.

Utilizing the analytical expression for the agent values given by equation (3.29) and the fact that idiosyncratic location tastes ε_i are drawn from Frechet distribution, we can derive $\bar{\mathcal{U}}^T(t)$ as follows[14]

$$\bar{\mathcal{U}}^T(t) = \Gamma\left(1-\frac{1}{\xi}\right)\frac{1}{\rho-g}\left[\sum_{i=1}^{N}\left(A_i C_i^T(t)\right)^{\xi}\right]^{\frac{1}{\xi}} \tag{3.39}$$

The derivation of this expression depends on the convenient properties of the Frechet distribution, and can be found in Appendix A.10. It should be noted that this expression is independent of migration frequency parameter ζ, since the function $\bar{\mathcal{U}}^T(t)$ represents the value of agents independent of their initial locations. Secondly, the welfare of agents increase with the aggregate growth rate of the economy, as higher growth implies higher consumption in the future. Finally, $\bar{\mathcal{U}}^T(t)$ can be considered as a weighted average of location specific consumption rates, weights being the amenities in locations A_i. The planner cares about an aggregate consumption across all locations, however, consumption in high amenity locations are valued relatively more.

The final social welfare function is defined as a weighted average of worker and inventor welfares,

[14]This derivation implicitly assumes that the growth rate of the economy g always stays lower than the time discount rate of ρ. Otherwise, agent value function explodes to infinity as the future consumption growth rate is higher than the discount rate.

where weights are chosen by the planner. It is defined as follows

$$\begin{aligned}\mathcal{W}(t) &\equiv \phi^L \bar{L} \times \bar{\mathcal{U}}^L(t) + \phi^R \bar{R} \times \bar{\mathcal{U}}^R(t) \\ &= \Gamma\left(1-\frac{1}{\xi}\right)\frac{1}{\rho-g}\left\{\phi^L \bar{L}\left[\sum_{i=1}^{N}\left(A_i C_i^L(t)\right)^{\xi}\right]^{\frac{1}{\xi}} + \phi^R \bar{R}\left[\sum_{i=1}^{N}\left(A_i C_i^R(t)\right)^{\xi}\right]^{\frac{1}{\xi}}\right\}\end{aligned} \tag{3.40}$$

The welfare weights for different agent types are given by ϕ^L and ϕ^R such that $\phi^L + \phi^R = 1$. In the rest of the paper, these weights are taken equal to each other, i.e. $\phi^L = \phi^R = 0.5$.

The planner maximizes the social welfare function (3.40) by choosing location specific R&D subsidy rates $s_i \in [0,1]$ subject to equilibrium condition in BGP. That is, the planner solves her problem in a constrained environment with a single policy tool available to her, place-based R&D subsidy rates. Taxation of R&D expenditures are not considered as a policy tool. In order to simplify the analysis, the tax rate chosen to finance the cost of the policy is set uniformly across all the locations. That is, while subsidy rates are location specific, labor income tax rate τ is uniform across locations. As the knowledge spillovers between locations has a nonlinear form given by (3.38), I solve planner's problem numerically.

4 Quantification

The model parameters are quantified with a combination of three steps. First of all, several aggregate parameters that are common to all locations are externally calibrated. Secondly, knowledge network represented by the matrix Ω is estimated from patent citation flows and citation lags between US states, which are the geographic unit of the analysis. Finally, I use the model to recover the remaining location specific parameters—fundamental research productivity $\bar{\alpha}_i$, and amenity A_i—from data on worker and inventor allocations in the US. Another aggregate parameter, entry cost f, is recovered from exactly matching model implied entry rate and the data counterpart. The method, which I call model inversion, infers location specific parameters that deliver worker and inventor location choices across US states as equilibrium outcomes.

The main intuition behind the estimation procedure outlined above is based on two important predictions of the model, given by equations (3.33) and (3.38). The first equation states that equilibrium level of endogenous research productivity of locations, a combination of exogenous factors and knowledge spillovers from other locations, can be inferred from relative ratio of inventor-to-worker fractions across locations. This result depends on the main assumption of the model—both inventors and workers value location amenities identically. After controlling for observed distribution of workers across US states, the remaining variation in inventor allocation identifies other factors that only affect inventors in their migration decisions, i.e. inventor wages. It should be noted that the implication of the simple structure of the model that workers earn the same wage in each location does not alter this line of reasoning. Even if worker wages were heterogeneous in a more complex model with location and worker specific productivity differences across locations, such a structural model would have allowed us to control for them via the corresponding migration decisions. The important point in this type of analysis is to have a structural model that would explain heterogeneous effects of locations on the earnings of different types of agents in the economy—workers and inventors. The model is intentionally kept simple for worker earnings characteristics so that the main intuition for the identification of heterogeneous research productivity of locations is more explicit.

The second equation (3.38) describes knowledge flows between locations. The main ingredient of this equation is diffusion rate parameters ω_{ij} which are specific to each state-pair. These parameters are inferred from patent citations and citation lags between state-pairs. The main assumption that justifies this exercise is a strong one, which is patent citations, although not perfect, reflect intertemporal knowledge spillovers in the innovation process. Citing inventors cite previous inventions from which they learn and inspire, and on which build. Thus, availability of this knowledge, K_i, increases their research productivity, as in the model.

Table 1: Externally calibrated parameters

Parameter	Description	Value	Source
ρ	Time discount rate	0.05	Matching 5% annual real rate
β	Labor share in production	0.6	Labor share
λ	Innovation step size	1.15	General literature
θ	Curvature of innovation function	2	General literature
δ	Idea obsolescence rate	0.075	Caballero and Jaffe (1993)
ξ	Location taste dispersion	2	Desmet et al. (2018)
φ	Share of past knowledge in research prod.	0.5	Externally set
$\bar{L}$	Total mass of workers	1	Normalization
s_i	R&D subsidy rate	0	Externally set

Externally calibrated parameters. Table 1 gives the list of externally calibrated parameters and the corresponding values. Except place-based R&D subsidies, which are taken to be zero, none of externally calibrated parameters vary across locations. One of the important parameters in this list is location taste dispersion (ξ) which directly affects endogenous sorting of agents into locations. In particular, although it does not alter the ranking of locations for estimated parameters, dispersion parameter shapes the concentration of agents in equilibrium. Lower values for ξ means that agents have more dispersed preferences for locations, hence in equilibrium less concentration arises. Another important parameter is φ that governs the importance of intertemporal knowledge spillovers in the innovation process relative to other location-specific exogenous factors. Higher φ corresponds to a higher share of past knowledge in the creation of future inventions. In what follows, this parameter is taken to be half, i.e. $\varphi = 0.5$.

The time frequency of the model is taken to be a year. As agents have linear preferences over time, the equilibrium interest rate equals to ρ which is taken to be 5% (annual), which is common in endogenous growth literature. Innovation step size $\lambda = 1.15$ lies in the range of several estimates in the literature. This parameter mainly affects the growth rate of the economy suggested by equation (3.23). β is taken to be 0.6 which is in line with an average labor share of 60% for the period studied. Innovation curvature parameter θ affects the marginal cost of innovation through the curvature of R&D production function with respect to researchers employed. In other types of growth models in which individuals are sorted between production and R&D, the curvature parameter has a direct effect on the aggregate growth rate of the economy through allocation of total labor into research activity. However, in this model, the total supply of inventors is assumed fixed, therefore, such implications are absent. Finally, exogenous rate of idea obsolescence δ is taken from Caballero and Jaffe (1993) where they estimate a similar citation equation given by (4.1) (will be explained below) in order to estimate the extent of intertemporal spillovers between time periods. A value of $\delta = 0.075$ implies an idea obsolescence rate of 7.5% in a year. The sole magnitude of this variable has a direct effect on the growth rate of the economy as higher obsolescence rate reduces the effectiveness of past ideas on research productivity and growth.

It should be noted that the parameters that affect the aggregate growth rate of the economy only alters the overall level of location-specific research productivity estimates. For instance, a higher δ implies lower growth all else equal. In order to match the constant growth rate of 1.37%, model inversion results in higher level of location specific research productivities without altering relative research productivities across locations. A more detailed discussion on the identification of relative research productivities can be found below.

4.1 Estimation of knowledge network

In this section, I derive an equation of citation probabilities across locations exploiting the idea diffusion structure of the model. This equation is labelled as citation equation in the rest of the paper, and estimated from patent citations data. We start with a thought experiment by asking what is the probability of a patent issued in j at time s being cited by patents issued in i at a later date $t \geq s$. First of all, ideas become obsolete over time with a rate of δ. Assuming that every patent is embedded with an idea intensity of one (normalization), the number of useful ideas remaining in the patent by time t is given by $e^{-\delta(t-s)}$. This is the average fraction of ideas that remain from time s to time t under the assumption that ideas are subject to independent obsolescence shocks with a rate of δ per unit of time. Secondly, I assume that inventors in location i at time t cite the patent if and only if they observe the idea in their location by time t. Equivalently, the necessary and sufficient condition for citation is the diffusion of the idea from j to i between time points s and $t \geq s$. Under the assumption of exponential distribution of diffusion lags, the probability that an idea diffuses from j to i by time t is given by $1 - e^{-\omega_{ij}(t-s)}$. As diffusion and obsolescence are independent events, the probability of citation is given by the product of two probabilities, i.e. $e^{-\delta(t-s)} \left[1 - e^{-\omega_{ij}(t-s)}\right]$.

Citation probability is affected by a number of factors. All else equal, the time lag has two opposing effects on citation probabilities. Citation probability is negatively correlated with time lag $t - s$ because of idea obsolescence channel. As ideas age older, the probability that idea stays useful by time t declines (the first term). On the other hand, citation probability increases by the time lag, as it is more likely for ideas to be diffused to other locations as more time passes since their invention (second term). Other factors are due to parameters δ and ω_{ij}. All else equal, higher rate of obsolescence decreases citation probabilities between all location pairs uniformly. Finally, as idea diffusion rate ω_{ij} increases between locations, then it becomes more likely that patents from the destination location i cites past patents that originated from the origin location j in a fixed time interval of length $t - s$.

Under these assumption we can derive the maximum time lag at which patent citation probability is maximized. Taking first order condition of $\max_\tau e^{-\delta\tau}\left[1 - e^{-\omega_{ij}\tau}\right]$ with respect to τ yields a location pair specific time lag $\tau^\star_{ij}$ at which citation probability from i to j is maximized as follows, $\tau^\star_{ij} = \frac{1}{\omega_{ij}} \ln\left(\frac{\omega_{ij}+\delta}{\delta}\right)$. $\tau^\star_{ij}$ decreases with ω_{ij} which implies that the peak citation probability is reached earlier as the rate of diffusion between locations rises. This observation forms the basis for the identification of diffusion rates from patent citation lags.

In order to derive an estimating equation of diffusion rates, I augment citation probability with location-time fixed effects separately for citing and cited locations. Let $\Gamma^{\text{citing}}_{it}$ denote the fixed effect for citing location i at time t which represents the technology composition of citing patents issued at time t in i. Similarly, $\Gamma^{\text{cited}}_{js}$ denotes the fixed effect for the technology composition of cited locations. These fixed effects aim to control for citing-cited technology composition of patent portfolios and its effect on the level of citation probabilities. The identification of ω_{ij} comes mainly from the citation lags. With the inclusion of fixed effects, the estimating equation becomes

$$\frac{C^{ts}_{ij}}{P_{it}P_{js}} = \Gamma^{\text{citing}}_{it} \times \Gamma^{\text{cited}}_{js} \times e^{-\delta(t-s)}\left[1 - e^{-\omega_{ij}(t-s)}\right] \tag{4.1}$$

The left hand side of equation (4.1) denotes the estimated patent citation probability which is defined as the observed number of citations from i's patents in time t to j's patents issued in time s (C^{ts}_{ij}) divided by the total number of all possible combinations between these two groups of patents, i.e. the product of the number of patents that are issued at time t in i (P_{it}) and the number of patents that were issued at time s in j (P_{js}). This equation is just equal to Caballero and Jaffe (1993) and Cai et al. (2022)'s citation equations, the only difference being it is modified in terms of citing-cited locations.[15] Equation (4.1) is estimated by nonlinear least squares with an iterative minimization procedure by fixing the obsolescence parameter $\delta = 0.075$ as estimated by Caballero and Jaffe (1993).

4.2 Model inversion

Equipped with the estimates of state-pair diffusion rates ω_{ij} from patent citation lags, we can finalize the quantification of the model with an inversion process by which location specific parameters, $\bar{\alpha}_i$ and A_i, are recovered. Table 2 shows the target moments from the data used, and the corresponding identified parameters. First of all, worker allocation across US states is targeted in order to pin down

[15] In Caballero and Jaffe (1993), citing and cited fixed effects are included for time periods t and s in order to capture different number of ideas generated in these time periods and compositional differences as discussed above. They mainly focus on intertemporal spillovers between time periods by estimating a single diffusion rate parameter. In this paper, I estimate location-pair diffusion parameters by adding a spatial aspect to their citation equation.

Table 2: Targeted moments and identified parameters

Target	Target Notation	Identified Parameter	Parameter Notation
1. Allocation of workers across locations	$\gamma_i^L,\ i = 1,\ldots,N$	(Relative) amenity	$A_i,\ i = 2,\ldots,N$ with $A_1 = 1$
2. Allocation of patenting inventors across locations	$\gamma_i^{R\star} = \gamma_i^R,\ i = 1,\ldots,N$	(Relative) exogenous research prod.	$\bar{\alpha}_i,$
3. Total number of patenting inventors	$\bar{R}^\star = \frac{F^2+1}{F^2+F} z\bar{R}$	Total number of inventors	$\bar{R}$
4. Aggregate entry rate	$\tilde{z}$	Entry cost	f
5. Aggregate growth rate	$g = \frac{1-\beta}{\beta}\log(\lambda)x$	Level of exogenous research prod.	$\bar{\alpha}_1$

amenity distribution across locations. The mapping between the two is given by equation (3.31). Denoting the number of locations by N, we have $N-1$ many moments to match, as the sum of fractions of workers across states adds up to one. Therefore, we can recover location amenities only up to a scale. After normalizing the amenity in the first location to be one, i.e. $A_i = 1$, the equation (3.31) implies

$$A_i = \left(\frac{\gamma_i^L}{\gamma_1^L}\right)^{\frac{1}{\xi}}$$

Higher relative worker share in a location suggests higher level of amenities in the location, as worker wages are equalized across locations. Thus, the only heterogeneity remaining in worker migration decisions stems from location specific characteristics A_i.

Secondly, observed inventor allocation across US states is utilized to estimate fundamental research productivities of locations, $\{\bar{\alpha}_i\}_{i=1}^N$, only up to a scale. As discussed above, observed number of inventors in the patent data cannot be directly mapped to the number of inventors in the model, as not all inventors apply for a patent in a given year. To back out the true inventor allocation from the data, I utilize the model's predictions on innovation probabilities.

In the model, the rate of probability of patenting (or innovation) per product line is given by zdt, where dt is the length of the time interval considered, which is taken as one year. As per product inventor employment in location i is given by r_i, the number of inventors that come up with a new invention per product line in a time interval of dt is equal to $zdt \times r_i$. Similarly, $\tilde{z}dt \times \tilde{r}_i$ many inventors who are employed by entrants are successful in patenting. Denoting the total number of successful inventors in i as $R_i^\star$, and taking $dt = 1$, we have

$$R_i^\star = \psi_i z r_i + \tilde{\psi}_i \tilde{z} dt \tilde{r}_i = \left(1 + \frac{1}{F^2}\right) z \psi_i r_i$$

From inventor market clearing (3.24), we also have

$$R_i = \psi_i r_i + \tilde{\psi}_i \tilde{r}_i = \frac{F+1}{F} \psi_i r_i \implies \psi_i r_i = \frac{F}{F+1} R_i$$

Substituting this expression into the first one yields a relationship between $R_i^\star$ and R_i such that

$$R_i = \frac{F^2 + F}{F^2 + 1} \frac{1}{z} R_i^\star \implies \bar{R} = \frac{F^2 + F}{F^2 + 1} \frac{1}{z} \bar{R}^\star$$

where $\bar{R}^\star$ is the number of successful inventors nationwide, and the second expression follows from the summation of the first across all locations. Therefore, for a given estimate of $F = f^{\theta/(\theta-1)}$ and model implied z, we can map observed $\bar{R}^\star$ to the unknown $\bar{R}$. Lastly, the fraction of all inventors located in i in the model, γ_i^R, equals to i's share of successful inventors, $\gamma_i^{R\star} \equiv R_i^\star / \bar{R}^\star$. This can be seen by dividing both equations to each other which gives rise to $\gamma_i^R = \gamma_i^{R\star}$. Note that $\gamma_i^{R\star}$ and $\bar{R}^\star$ are observed from the patent data.

Fundamental research productivities $\bar{\alpha}_i$ are recovered in two steps. Firstly, equation (3.33) is used to back out endogenous relative productivities α_i, after replacing γ_i^R with $\gamma_i^{R\star}$. The intuition is explained as before, i.e. after controlling for amenity differences by the observed worker allocations, we can then estimate other factors that alter inventor earnings across locations. These factors are captured by location level resources for R&D and innovation. Secondly, we can use the definition $\alpha_i = \bar{\alpha}_i^{1-\varphi} K_i^\varphi$ and the endogenous formation of K_i across locations using model implied location innovation rates. In particular, an iterative procedure is employed using equation (3.38) given estimates of α_i to recover exogenous research productivities across locations $\bar{\alpha}_i$ up to a scale.

The aggregate entry rate in the model is equal to $\tilde{z}$, as all potential entrants choose the same innovation rate no matter where they are located. Relevant entry rate for the period analyzed is 9%, taken from Akcigit and Ates (2023). Entry cost parameter is pinned down by matching the entry rate with the data. Finally overall level of $\bar{\alpha}_i$ is recovered by matching the model implied growth rate and its data counterpart which is taken to be 1.37% from Akcigit and Ates (2023). High absolute level of research productivity increases the frequency with which inventors come up with new ideas, thus increasing the growth rate of aggregate productivity $\mathcal{A}(t)$.

5 Results

In this section, I present estimation results and resulted optimal place-based R&D policies in two stages. In the first stage, I assume that the US economy is comprised only of the top 10 states in terms of patenting. The reasons I focus on these states are three folds. Firstly, it is easier to discuss estimation

Table 3: Top 10 states in patenting

State code	State name	Patent share in 2005
CA	California	25%
TX	Texas	7%
NY	New York	6%
MA	Massachusetts	5%
WA	Washington	4%
MI	Michigan	4%
IL	Illinois	4%
NJ	New Jersey	4%
MN	Minnesota	4%
PA	Pennsylvania	3%
	Total	65%

results and the resulted optimal policy with fewer locations. Secondly, these states are well-known as being innovation locomotives of the US. Lastly, patent citation flows are the most intensive among these regions. In order to draw conclusions on the effect of knowledge network on parameter estimates, I estimate two versions of the model. The first one is performed assuming $\varphi = 0$, i.e. the knowledge spillovers across regions are shut down. The second estimation is performed for the baseline model in which knowledge spillovers are active, $\varphi = 0.5$. After comparing estimation results, I proceed with the implied optimal policy for both estimations. Finally, I do several counterfactual exercises in order to assess the importance of knowledge spillovers and amenities for the characterization of optimal policy. In the second stage, I perform the estimation for the whole US economy, i.e. 51 states including DC. Qualitatively similar effects of knowledge spillovers arise in the full estimation as in the case for top 10 states. I, then, solve for the optimal R&D policy and discuss welfare implications.

5.1 Results for top 10 patenting states

In this section, it is assumed that the US economy is comprised of only ten states that produced the most of patents in 2005. Total share of these states in aggregate patenting is 65%. California (CA) comes first with a share of 25% followed by Texas (TX) with 7% and New York (NY) with 6%. The smallest shares belong to New Jersey (NJ), Minnesota (MN) and Pennsylvania (PA) with respective shares of 4%, 4%, and 3%. Table 3 illustrates the huge concentration of patenting even among the top ten most innovative locations.[16]

Table 4 illustrates parameter estimates for the sample states. In this version of the estimation, the knowledge network is inactive, i.e. $\varphi = 0$. Endogenous research productivity α_i is exactly equal to location fundamentals measured by $\bar{\alpha}_i$. The most research productive state is estimated to be Washington with a research productivity of 1.8 times that of the least productive state Pennsylvania.

[16] Worker and inventor shares are recalculated among ten states. For instance, California, the state with the highest share of inventors, is home to 22% of all inventors in the US, while, among the top 10 states, its share increases to 35%. Other two aggregate targets, growth and entry rates, are kept same in their original values, 1.37% and 9%, respectively.

Table 4: Parameter estimates for top 10 states - Without knowledge spillovers

States		**Parameters**		**Allocations**	
Code	**Name**	**Prod.** $\bar{\alpha}$	**Amenity** A	**Inventors** γ^R	**Workers** γ^L
WA	Washington	0.48	0.42	0.08	0.04
MA	Massachusetts	0.42	0.48	0.08	0.06
CA	California	0.42	1.00	0.35	0.24
MN	Minnesota	0.38	0.43	0.05	0.04
MI	Michigan	0.34	0.54	0.07	0.07
NJ	New Jersey	0.32	0.52	0.06	0.07
NY	New York	0.29	0.75	0.09	0.14
TX	Texas	0.29	0.79	0.10	0.15
IL	Illinois	0.28	0.63	0.06	0.10
PA	Pennsylvania	0.27	0.62	0.06	0.09

Note: Rows are ordered from the highest $\bar{\alpha}$ to the lowest

In terms of amenities, California has the highest, while Washington has the least, less than half of California. Although California has the highest fraction of inventors among these states, it also has the largest employment share with 24%. For the identification of research productivities across locations, number of inventors alone is not informative as inventors also value location amenities. As discussed in Section 4.2, the ratio of inventor share to worker share is the moment that identifies research productivities. As an example, the most research productive state Washington is home to 8% of inventors with an employment share of 4%. The ratio of the two is higher than that for California. Another example is the second largest state in terms of inventor count, Texas. 10% of inventors among ten states are observed to locate in Texas, while 15% of employment takes place in there. Thus, inventors choose Texas relatively less frequently than workers, informing the model inversion about relatively lower research productivity in Texas. On the contrary, amenities are directly identified by worker shares putting Texas on the second place in terms of relative amenities.

When knowledge spillovers are allowed between locations, the ranking of states in terms of research productivities change significantly. In order to proceed with model inversion, first knowledge diffusion rates ω_{ij} are estimated from the patent citation (4.1). Figure 7 depicts the estimated network matrix Ω in a heatmap plot. In this figure, origin states, represented as columns, refer to states from where ideas diffuse to the rest. Destination states, represented as rows, are the states to where ideas diffuse from origins. Some observations are in order. Firstly, diagonal terms have the highest values suggesting that within location spillovers are stronger than spillovers across different states, in line with the findings of Jaffe et al. (1993). Secondly, knowledge network Ω is observationally a symmetric matrix. For instance, CA exports knowledge mostly to WA, TX, PA, NY, and NJ (CA column), and it also imports knowledge mostly from these states (CA row). Thus, we can conclude that if i is connected to j, it is likely that j is also connected to i. Connections between states are mostly bilateral. Thirdly, as can be inferred from columns, California, New York, and New Jersey are the most upstream states in the flow of ideas. That is, these states export ideas relatively faster than other states. Lastly, Minnesota (MN)

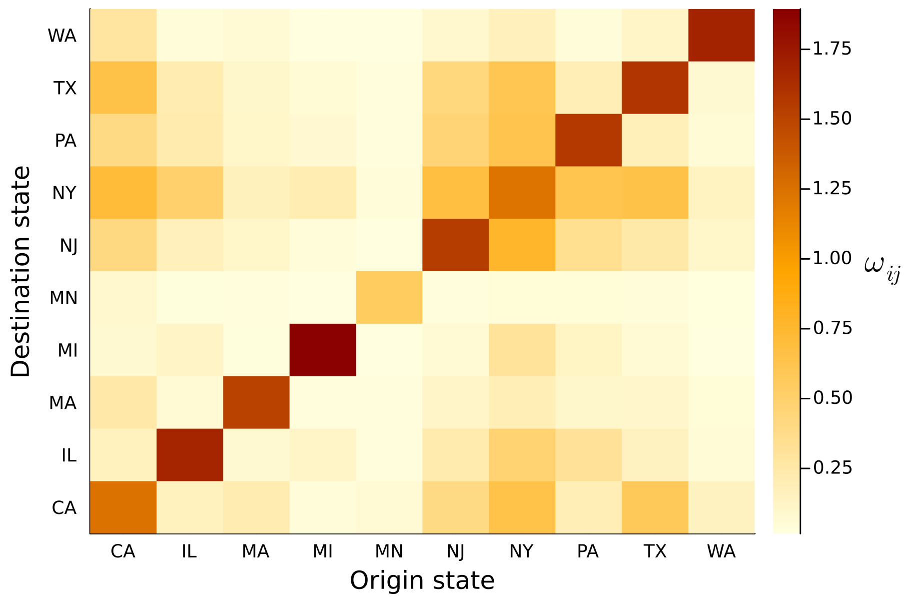


Figure 7: Estimated knowledge network Ω for ten states

stands alone as being the least connected state both in terms of idea exports and imports.

Table 5 shows parameter estimates based on the estimated Ω matrix. Amenity estimates are same as before, however, inclusion of knowledge network to estimation alters the estimates for $\bar{\alpha}_i$. In particular, Minnesota rises to second place in terms of research productivity while California declines to fifth place. The reason is that observed number of inventors in Minnesota (relative to its workers) can only be rationalized with a high $\bar{\alpha}_i$ estimate as Minnesota stands alone as the state that benefits the least from knowledge spillovers. Similarly, overall research productivity α_i in California mostly stems from network effects so that in terms of exogenous research productivity $\bar{\alpha}_i$, California declines from third to fifth place.

Table 5: Parameter estimates for top 10 states - With knowledge spillovers

States		**Parameters**	
Code	**Name**	**Prod.** $\bar{\alpha}$	**Amenity** A
WA	Washington	0.18	0.42
MN	Minnesota	0.17	0.43
MA	Massachusetts	0.14	0.48
MI	Michigan	0.12	0.54
CA	California	0.11	1.00
NJ	New Jersey	0.07	0.52
IL	Illinois	0.06	0.63
TX	Texas	0.06	0.79
NY	New York	0.05	0.75
PA	Pennsylvania	0.05	0.62

Note: Rows are ordered from highest $\bar{\alpha}$ to lowest

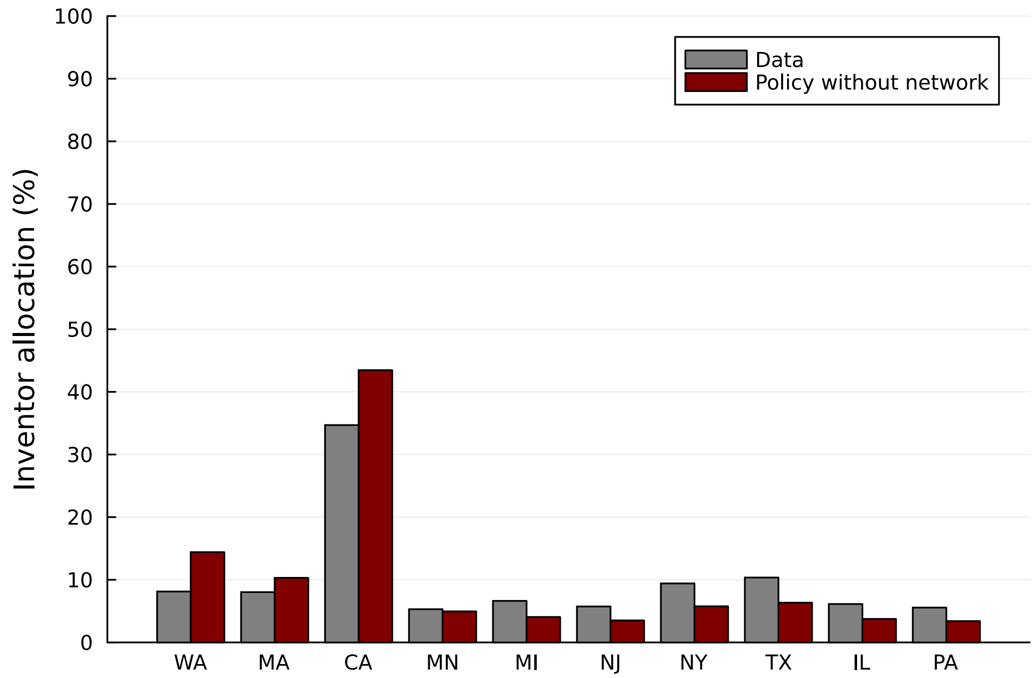


Figure 8: Inventor allocation under optimal policy - Without knowledge spillovers

Note: States are ordered left-to-right from the highest $\tilde{\alpha}$ estimate to the lowest under "without" spillovers estimation.

Optimal policy. The change in estimated allocation of exogenous research productivity across locations has implications on the optimal policy. In an environment without knowledge spillovers between locations, optimal policy only corrects the dispersion in idiosyncratic location preferences of inventors. In decentralized equilibrium without spillovers, there is always positive measure of inventors who idiosyncratically value the least research productive state the most among all, although they earn a very low wage there. They compensate the low productivity (and resulted low earnings and consumption) with their private value for the location. However, this is not aligned with the objectives of the planner, as the planner also cares about the effect of inventors on the rate of economic growth. Thus, optimal policy aims to relocate researchers towards the most research productive states by place-based R&D subsidies. Relocation of all of the inventors to the most productive state is extremely costly in terms of the forgone consumption due to taxation, as Frechet taste distribution has heavy tails. The trade-off that the planner faces is the tension between higher consumption in the future due to higher output growth, and lower current consumption due to taxation.

Figure 8 shows the inventor allocation under optimal policy for the case without knowledge spillovers (maroon bars) compared to the observed allocation in the data (gray bars). In order to achieve the optimal allocation, the planner subsidizes R&D expenditures only in four states: WA by 41%, MA by 31%, CA by 30%, and finally MN by 19%. This policy is financed by a permanent 1.5% uniform labor income tax. Under optimal policy, inventor allocation is more concentrated towards the most three productive states, WA, MA and CA. An interesting case is Minnesota (MN). Although it is subsidized, Minnesota experiences a decline in its inventor share under the optimal policy. That is, inventor share of MN would have been lower without subsidies. Optimal policy causes a welfare increase of 0.47% in consumption equivalent terms, while the long run growth rate of the economy rises from 1.37% to 1.41%. This exercise verifies the main intuition behind the workings of the optimal policy. Although there are no knowledge spillovers in this version of the model, the planner still wants to correct for the dispersion in idiosyncratic location tastes across inventors, as explained above. As spillovers are absent, the welfare gain from the policy is moderate.

When knowledge spillovers are present, the optimal policy starts reacting to the linkages between locations. In the baseline model, inventors do not internalize their effect on the productivity of other inventors through knowledge spillovers. While original incentive of the planner to relocate inventors to most productive states is still operating, spatial linkages makes the policy nontrivial. The trade-off that the planner faces is between research productivity and network centrality of locations. It might not be optimal for the planner to relocate inventors to most productive states if those locations are not connected well with the rest of the geography. Instead, it might be a better strategy to relocate inventors to moderately productive states but with strong linkages to the rest, both in upstream and downstream sense. By this way, the planner maximizes the extent of knowledge spillovers. It should be also noted that research productivity and growth considerations are not only factors that shape the optimal policy. Relative amenities directly affect the social welfare function (3.40). The effect of amenities on optimal policy is through two channels. The first channel is the direct effect. All else equal, the planner wants to benefit from highest amenities in the country. The second channel is through the effect of amenities on the cost of the policy. If a location simultaneously have both high amenity and research productivity, then reallocation of inventors to that location would be less costly in terms of taxation, as inventors would be more likely to migrate to that location due to high amenities in there. However, if amenities and research productivities are not aligned well, then the planner has to subsidize R&D very heavily in order to be able to convince inventors to migrate there. This increases the taxes imposed, hance the cost of the policy.

In order to assess the discussed effects of the knowledge network on policy, place-based R&D subsidies are solved for the baseline model with knowledge spillovers and compared to the previous case. That is, optimal policy is solved for the parameterization given by Table 5. Figure 9 shows the inventor allocation under the optimal policy in this case (depicted by red bars) while comparing it to previous policy and the data. In this case, subsidy rate in WA and CA rises to 49% in each, whereas it declines slightly to 30% for MA. MN is not subsidized anymore. Qualitatively, the planner stops allocating inventors only to the most productive states. The most clear example is Minnesota (MN). In the new policy, Minnesota experiences a stark decline in its share of inventors. The reason is that it is not well connected within the knowledge network (as suggested by the Ω matrix in Figure 7), so the social value of high research productivity in Minnesota decreases as other locations do not benefit much from the spillovers from Minnesota. Similarly, Massachusetts (MA) also experiences a decline in its inventor share. In the new policy, inventors are relocated towards WA and CA mostly from MA and MN.

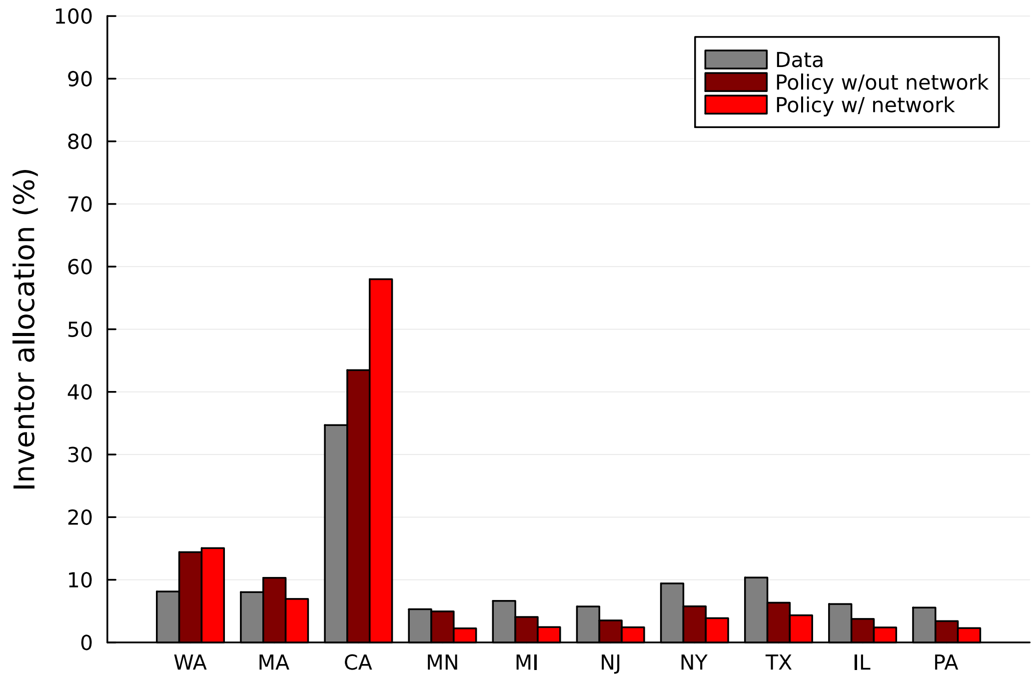


Figure 9: Inventor allocation under optimal policy - With knowledge spillovers

Note: States are ordered left-to-right from the highest $\bar{\alpha}$ estimate to the lowest under "without" spillovers estimation.

Inventor concentration across locations under the new policy results in higher than before. In addition to the network effects discussed above, another reason for this result is that knowledge spillovers are the strongest within states (high diagonal elements of Ω). Therefore, inventors cause an agglomeration type of spillovers in their own locations, which calls for more concentration under optimal policy. Finally, welfare increase in a model with knowledge spillovers is found to be higher, i.e. 1.68% consumption equivalent increase in welfare with a growth rate of 1.49%. Although the size of welfare effects are not directly comparable between two models, intuitively, we can argue that knowledge spillovers and resulting increasing returns to scale makes the optimal policy more effective in terms of welfare increases. In order to draw more meaningful comparison between the two policies, i.e. one that respects the knowledge spillovers between locations, and the other that does not consider linkages, we can implement the first policy within the estimated model with knowledge network. This exercise results in a welfare increase of 1.20% in consumption equivalent terms, which is 0.48 percentage points lower than the welfare increase under the policy that respects the knowledge spillovers.

5.2 Results for all states

In this section, I present estimation results of the model with all the states in the US (51 states including DC). Then I compare model implied untargeted moments with the data in order to validate parameter estimates. Finally, I solve the optimal place-based R&D subsidy policy. My findings can be

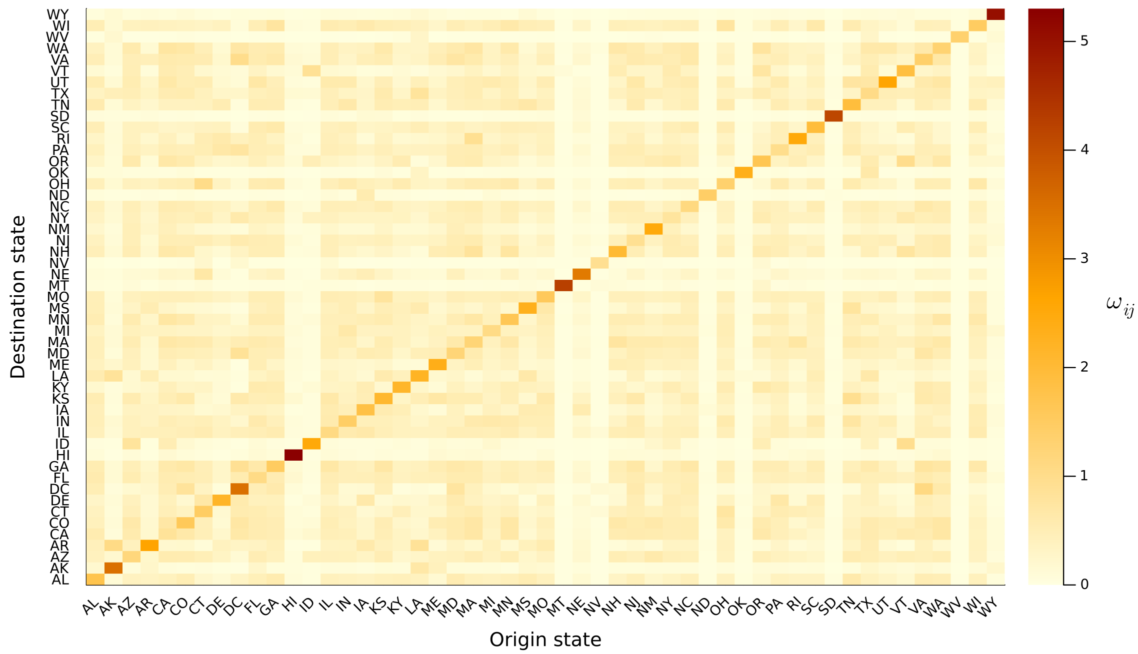


Figure 10: Estimated knowledge network Ω

summarized as follows. Model fit to untargeted moments is good giving confidence on the model's validity. Optimal policy calls for concentration of inventors in a few states on both West and East coasts such as Washington, Massachusetts, California, and Vermont. Social welfare increases 1.8% in consumption equivalent terms as a result of the proposed place-based R&D policy. Increase in welfare is associated with a 0.14 percentage points increase in annual growth rate of the economy, from 1.37% to 1.41%.

Estimated knowledge network Ω. The heatmap of estimated Ω matrix is given by Figure 10. Similar results are observed in the full estimation of the matrix. First of all, diagonal elements are considerably higher than off-diagonal elements suggesting strong within location spillovers. It can be argued that strength of connections between states are usually bilateral. Some states are isolated from the rest of the network both in upstream and downstream sense, such as Idaho, Montana, South Dakota, West Virginia. The (unweighted) mean of ω_{ij} is 0.31 suggesting a lag of 3.2 years in idea diffusion across states. The histogram of estimates for diffusion rates ω_{ij} is plotted in Figure 11 suggesting a bimodal distribution of pairwise diffusion rates across US states.

In order to test the validity of ω estimates, I regress estimated ω_{ij} on some observed characteristics of state pairs. These are pairwise physical distance between i and j, and academic citation shares, migration flows, number of air passengers, and trade flows, both from i to j, and from j to i. Only the coefficient estimates of physical distance and academic citation shares are significant, and their signs are as expected. In words, estimated ω_{ij} decreases in physical distance between i and j. Moreover, it is positively correlated with probability of academic papers published in i citing papers from j, and

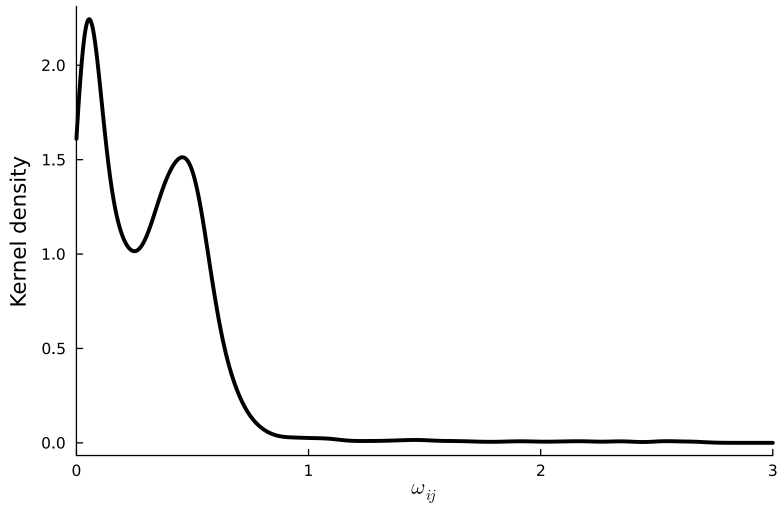


Figure 11: Kernel density of diffusion rate estimates

vice versa. Binscatter plots for the relationship between ω_{ij} estimates and observed characteristic are shown in Figure 12.

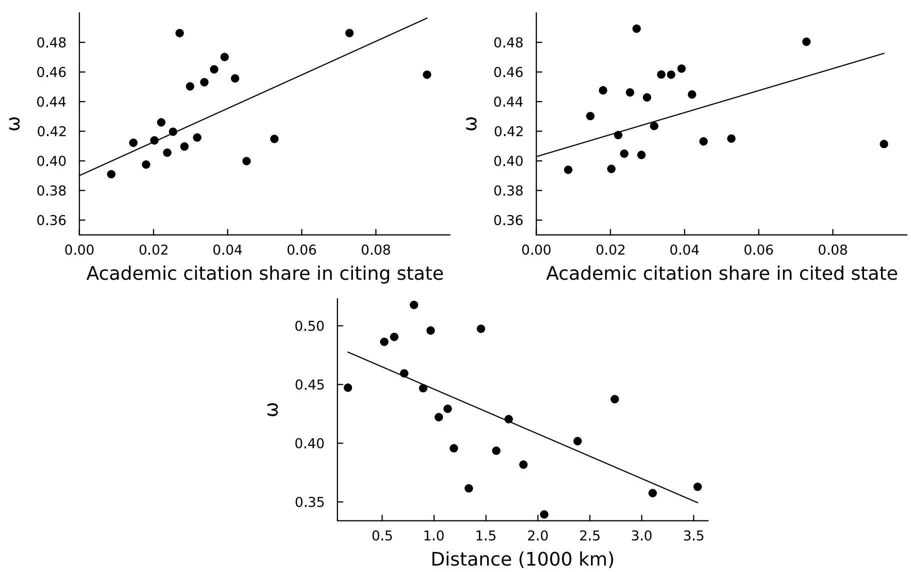
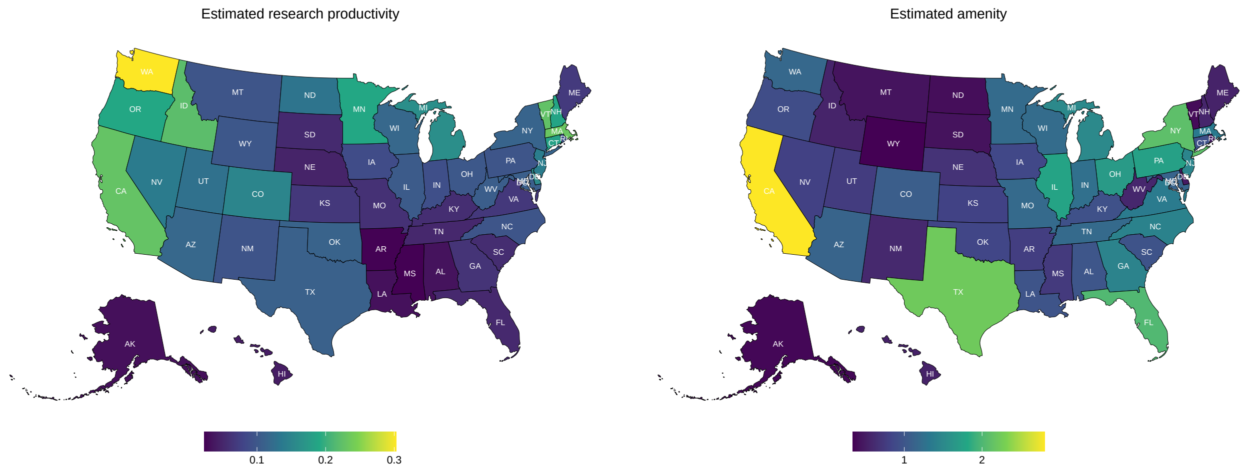


Figure 12: Estimates of ω_{ij} and their correlates

Location specific parameter estimates. Figure 13a illustrates the distribution of estimated research productivity $\bar{\alpha}$ across US states. The most research productive state is estimated to be Washington with a value of $\bar{\alpha} = 0.303$ followed by Massachusetts and California. The least research productive state is Mississippi with a value of $\bar{\alpha} = 0.024$. The mean of $\bar{\alpha}$ estimates is 0.110, and their standard deviation is 0.063. As is clear from Figure 13a, physical proximity is an important determinant of the spatial distribution of $\bar{\alpha}_i$. That is, closer states are also similar in terms of research productivity. West



(a) Research productivity $\bar{\alpha}_i$ (b) Amenity A_i

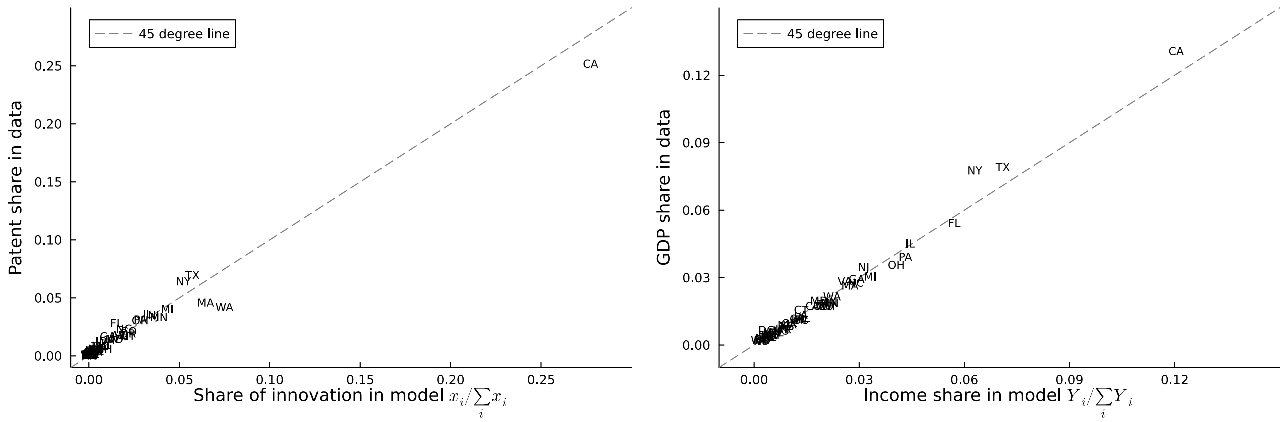
Figure 13: Estimated location specific parameters

coast states represented by Washington, Oregon, California, and perhaps including Idaho are most productive states in research along with a clustering on the East coast represented by Massachusetts, Vermont, Connecticut, and New Hampshire. In the Midwest, Minnesota and Michigan stand out.

Amenities, on the other hand, do not seem to be correlated much with research productivities. California has the highest amenity estimate followed by Florida, New York, Texas, Illinois.

Model fit for untargeted moments. In this section, the model is tested in terms of its fit to untargeted moments such as relative patenting rates of states, GDP shares, share of states in total R&D expenditure, and finally, R&D intensity of states defined as the ratio of R&D expenditure to state's GDP. The model performs well along these dimensions giving confidence on the validity of the model. Rate of patenting in a state in the model is given by x_i. The share of state's patents produced in a given year in total number of patents produced in the US can be measured by $x_i / \sum_i x_i$. Figure 14a shows the comparison of this moment between model and the data. Most of the observations lies on the 45 degree line implying almost perfect match. Another untargeted moment is GDP share of states. The GDP of a state in the model economy is defined as the total income of agents (including profits) located in the state, $Y_i \equiv (1+d)\left[W_i^L(t)L_i + W_i^R(t)R_i\right]$. Share of state's GDP in total US GDP is given by Figure 14b showing a very good match between model and the data. States vary slightly around the 45 degree line.

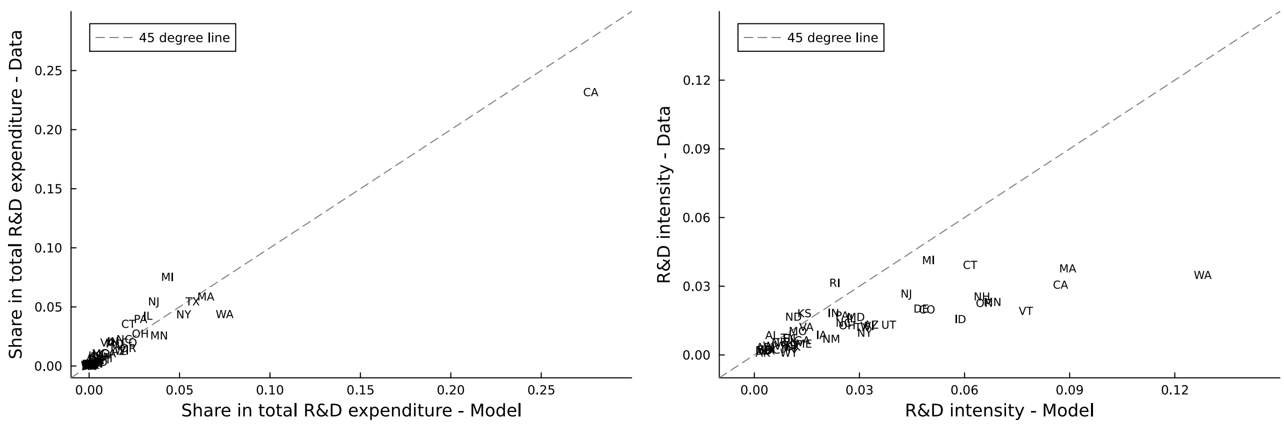
Finally, I check implied state level R&D expenditures from the estimated model, and compare it to the data obtained from National Science Foundation's (NSF) National Patterns of R&D Resources for the year 2004. This data provides state level R&D expenditures of private industry and government. Only industry R&D expenditures are included in state level R&D spending. In the model, state level R&D expenditure is given as the total wage bill of researchers employed in both incumbent and entrant firms. It is equal to $W_i^R(t)R_i$, as R&D subsidy rates are taken to be zero in the benchmark estimation.



(a) State's share in patenting

(b) State's share in GDP

Figure 14: Model's fit to untargeted moments



(a) State's share in total R&D expenditure

(b) State level R&D intensity

Figure 15: Model's fit to untargeted moments

Figure 15a compares the share of state's R&D expenditures in the model and in the data. Again, the model fit is very good in terms of relative R&D spending across states. On the right panel, state level R&D intensities are plotted. R&D intensity of a state is defined as the ratio of R&D expenditures to state level GDP. The model predicts a higher level of R&D intensity for most of the states relative to the data. However, model implied moment and data counterpart are positively correlated.

5.3 Optimal policy

Under the optimal place based R&D subsidy scheme, only 9 states receive R&D subsidies. Most heavily subsidized state is Washington with 57%, and the least subsidized state is Colorado with 9%. Remaining states do not receive R&D subsidies. Table 6 lists the R&D subsidy rates across states from the highest to the lowest. On the West coast, neighbor states California, Oregon and Washington receive R&D subsidies as this region of the country is the most R&D productive, and they are relatively

Table 6: Subsidy rates under optimal policy

State code	State name	Subsidy s
WA	Washington	0.57
MA	Massachusetts	0.48
CA	California	0.48
VT	Vermont	0.39
MN	Minnesota	0.35
OR	Oregon	0.34
NH	New Hampshire	0.32
CT	Connecticut	0.24
CO	Colorado	0.09

connected with the rest of the states. Minnesota is the only state that is subsidized in the Midwest, while the small region around Massachusetts benefit from subsidies as well. In terms of inventor allocation under the optimal policy, only seven states out of nine increase their inventor share. These two states that experience a reduction in inventors are Connecticut and Colorado. The reason is that concentration of inventors under the optimal policy is strongly towards the most research productive states, but the planner does not want to relocate much from Connecticut and Colorado. By R&D subsidies the decline in these two states is mitigated.

The optimal policy calls for concentration of inventors in top productive states such as Washington, Massachusetts, and California. The percent change in number of inventors between the optimal policy and the data for each state is depicted in Figure 16. Except the top five states, the rest lose almost half of their inventors under the new allocation. Washington experiences a 169% increase in its inventors relative to the data. Its share rises from 5.2% to 14%. Massachusetts and California observe similar increase in their inventors, 79% and 77%, respectively. Under the new allocation, California still has the highest share of inventors with 39.2%. Among the losing states, small states lose the most. Table 7 shows the top 5 gaining and losing states in terms of inventor counts under the optimal policy.

The reason for increased concentration of inventors under the policy is close geographical connections between states and strong within state knowledge spillovers. Although there is no reduced form agglomeration spillovers in the model, high within diffusion rates ω_{ii} implies local inventors benefit most from the local spillovers. This induces the planner to relocate more to the most productive states. However, geographical proximity also seems an important determinant of new inventor allocation as diffusion rates and proximity are strongly correlated as shown previously.

The overall welfare increases 1.8% in consumption equivalent terms. This welfare increase is achieved even without allocating the labor force towards being researchers, rather it is due to geographical reallocation of a constant pool of inventors in the country. This policy exercise shows the importance of knowledge spillovers even inside a country between localities, and points to a significant level of welfare loss due to imperfect knowledge spillovers specific to the innovation process. The 1.8% increase in welfare is associated with a 0.14 percentage points increase in the growth rate of the economy. As inventors benefit more from knowledge spillovers, their research productivity rises

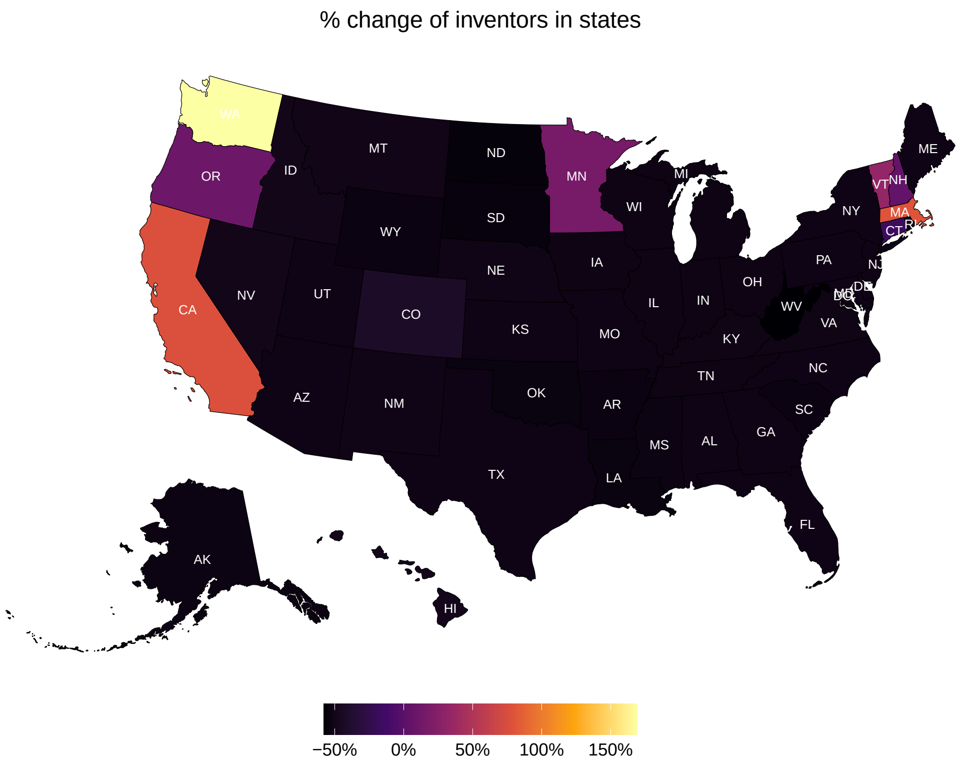


Figure 16: Percent change in number inventors under policy

helping them innovate more frequently. The growth rate rises from the targeted value 1.37% to 1.51% under the optimal policy.

Table 7: Top gaining and losing states

State code	**State name**	γ^R	$\gamma^{R\star}$	$100\frac{R^\star - R}{R}$
	Top 5 gaining states			
WA	Washington	5.2	14.0	+169%
MA	Massachusetts	5.1	9.2	+79%
CA	California	22.2	39.2	+77%
VT	Vermont	0.40	0.53	+33%
MN	Minnesota	3.4	3.9	+16%
	Top 5 losing states			
WV	West Virginia	0.14	0.06	-58%
ND	North Dakota	0.11	0.05	-56%
SD	South Dakota	0.06	0.03	-55%
LA	Louisiana	0.33	0.15	-54%
OK	Oklahoma	0.49	0.22	-54%

6 Conclusion

In this paper, I study the spatial allocation of inventors across US states and the effect of their location choice on the aggregate rate of innovation and growth, with a particular focus on knowledge spillovers across states. Empirically, it is shown that innovative activity is spatially concentrated more than other indicators such as employment and GDP. Furthermore, distribution of inventor to worker ratio across US states is highly right skewed suggesting that inventors prefer certain locations more than other workers. These locations coincide with the innovation hubs of the US such as California, Washington and the Northeast corridor. Finally, I show the extent of spatial concentration in patent citations, and argue that the variation in patent citation lags across citing-cited state pairs is particularly informative on the extent of knowledge linkages between them. These spillovers are intertemporal in the sense that future inventions benefit from the old ideas previously invented, which reflects itself as citations between patent documents.

On the theory side, a novel endogenous growth model is built with inventor and worker migrations in space, and mobile entrants who create firms in which inventors are employed for R&D purposes. The model is equipped with a knowledge diffusion network between locations which is estimated from patent citations. Inventors in the model do not internalize the effect of their location choice on the diffusion of ideas to other locations. Thus, the planner corrects for this externality by place-based R&D subsidies while taking into account heterogeneous linkages between US states.

Location specific parameters, amenities and exogenous research productivities, are recovered with a model guided inversion procedure which exactly matches observed worker and inventor allocations across US states. The unknown knowledge diffusion network is estimated from a patent citation equation that is derived from the model. It is shown that states that are close in distance are more likely to be connected, academic citation flows are positively correlated with the strength of connections, and within-location spillover rates are the highest. Based on all parameter estimates, the optimal placed based R&D subsidy policy is found as to maximize the social welfare function. The policy calls for even more spatial concentration of inventors although the model is absent from standard within-location agglomeration spillovers. Moreover, the policy respects the flow of ideas in space in the design of the place-based subsidy rates. The optimal policy increases the social welfare by 1.8% in consumption equivalent terms. The increase in welfare is associated with a 0.14 percentage points increase in the aggregate growth rate of the economy due to the maximum utilization of the knowledge spillover network between states as a result of the policy.

References

Acemoglu, Daron, Ufuk Akcigit, and William R Kerr, "Innovation network," *Proceedings of the National Academy of Sciences*, 2016, *113* (41), 11483–11488.

_ , _ , **Harun Alp, Nicholas Bloom, and William Kerr**, "Innovation, reallocation, and growth," *American Economic Review*, 2018, *108* (11), 3450–3491.

Aghion, Philippe and Peter Howitt, "A Model of Growth Through Creative Destruction," *Econometrica: Journal of the Econometric Society*, 1992, pp. 323–351.

Akcigit, Ufuk and Sina T Ates, "What happened to US business dynamism?," *Journal of Political Economy*, 2023, *131* (8), 2059–2124.

_ **and William R Kerr**, "Growth through heterogeneous innovations," *Journal of Political Economy*, 2018, *126* (4), 1374–1443.

_ **, Douglas Hanley, and Stefanie Stantcheva**, "Optimal Taxation and R&D Policies," *Econometrica*, 2022, *90* (2), 645–684.

Bloom, Nicholas, Mark Schankerman, and John Van Reenen, "Identifying technology spillovers and product market rivalry," *Econometrica*, 2013, *81* (4), 1347–1393.

Buzard, Kristy and Gerald Carlino, "The geography of research and development activity in the US," in "Handbook of Industry Studies and Economic Geography," Edward Elgar Publishing, 2013.

_ **, Gerald A Carlino, Robert M Hunt, Jake K Carr, and Tony E Smith**, "Localized knowledge spillovers: Evidence from the spatial clustering of R&D labs and patent citations," *Regional Science and Urban Economics*, 2020, *81*, 103490.

Caballero, Ricardo J and Adam B Jaffe, "How high are the giants' shoulders: An empirical assessment of knowledge spillovers and creative destruction in a model of economic growth," *NBER macroeconomics annual*, 1993, *8*, 15–74.

Cai, Jie, Nan Li, and Ana Maria Santacreu, "Knowledge diffusion, trade, and innovation across countries and sectors," *American Economic Journal: Macroeconomics*, 2022, *14* (1), 104–145.

Carlino, Gerald and William R Kerr, "Agglomeration and innovation," *Handbook of regional and urban economics*, 2015, *5*, 349–404.

Carrincazeaux, Christophe, Yannick Lung, and Alain Rallet, "Proximity and localisation of corporate R&D activities," *Research Policy*, 2001, *30* (5), 777–789.

Crews, Levi Garrett, "A dynamic spatial knowledge economy," 2023.

Davis, Donald R and Jonathan I Dingel, "A spatial knowledge economy," *American Economic Review*, 2019, *109* (1), 153–170.

Desmet, Klaus, Dávid Krisztián Nagy, and Esteban Rossi-Hansberg, "The geography of development," *Journal of Political Economy*, 2018, *126* (3), 903–983.

Duranton, Gilles and Diego Puga, "Micro-foundations of urban agglomeration economies," in "Handbook of regional and urban economics," Vol. 4, Elsevier, 2004, pp. 2063–2117.

Ellison, Glenn and Edward L Glaeser, "Geographic concentration in US manufacturing industries: a dartboard approach," *Journal of political economy*, 1997, *105* (5), 889–927.

Fajgelbaum, Pablo D, Eduardo Morales, Juan Carlos Suárez Serrato, and Owen Zidar, "State taxes and spatial misallocation," *The Review of Economic Studies*, 2019, *86* (1), 333–376.

Gaubert, Cecile, "Firm sorting and agglomeration," *American Economic Review*, 2018, *108* (11), 3117–3153.

Gerlach, Heiko, Thomas Rønde, and Konrad Stahl, "Labor pooling in R&D intensive industries," *Journal of Urban Economics*, 2009, *65* (1), 99–111.

Glaeser, Edward L, "Learning in cities," *Journal of urban Economics*, 1999, *46* (2), 254–277.

Gross, Daniel P and Bhaven N Sampat, "America, jump-started: World War II R&D and the takeoff of the US innovation system," *American Economic Review*, 2023, *113* (12), 3323–3356.

Grossman, Gene M and Elhanan Helpman, "Quality ladders in the theory of growth," *The review of economic studies*, 1991, *58* (1), 43–61.

Hall, Bronwyn H, Adam B Jaffe, and Manuel Trajtenberg, "The NBER patent citation data file: Lessons, insights and methodological tools," 2001.

Helsley, Robert W and William C Strange, "Innovation and input sharing," *Journal of Urban Economics*, 2002, *51* (1), 25–45.

Jaffe, Adam B, "Real effects of academic research," *The American economic review*, 1989, pp. 957–970.

_ , Manuel Trajtenberg, and Michael S Fogarty, "Knowledge spillovers and patent citations: Evidence from a survey of inventors," *American Economic Review*, 2000, *90* (2), 215–218.

_ , _ , and Rebecca Henderson, "Geographic localization of knowledge spillovers as evidenced by patent citations," *the Quarterly journal of Economics*, 1993, *108* (3), 577–598.

Kantor, Shawn and Alexander Whalley, "Knowledge spillovers from research universities: evidence from endowment value shocks," *Review of Economics and Statistics*, 2014, *96* (1), 171–188.

Klepper, Steven, "The origin and growth of industry clusters: The making of Silicon Valley and Detroit," *Journal of urban economics*, 2010, *67* (1), 15–32.

Klette, Tor Jakob and Samuel Kortum, "Innovating firms and aggregate innovation," *Journal of political economy*, 2004, *112* (5), 986–1018.

Kline, Patrick and Enrico Moretti, "Local economic development, agglomeration economies, and the big push: 100 years of evidence from the Tennessee Valley Authority," *The Quarterly journal of economics*, 2014, *129* (1), 275–331.

Krugman, Paul, *Geography and trade*, MIT press, 1992.

Lamoreaux, Naomi R, Margaret Levenstein, and Kenneth L Sokoloff, "Financing invention during the second industrial revolution: Cleveland, Ohio, 1870-1920," 2004.

Landier, Augustin, "Entrepreneurship and the Stigma of Failure," *Available at SSRN 850446*, 2005.

Liu, Ernest and Song Ma, "Innovation networks and r&d allocation," Technical Report, National Bureau of Economic Research 2021.

Marshall, Alfred, *Principles of economics: unabridged eighth edition*, Cosimo, Inc., 2009.

Moretti, Enrico, "The effect of high-tech clusters on the productivity of top inventors," *American Economic Review*, 2021, *111* (10), 3328–3375.

Nicholas, Tom and James Lee, "The origins and development of Silicon Valley," 2013.

Peri, Giovanni, "Determinants of knowledge flows and their effect on innovation," *Review of economics and Statistics*, 2005, *87* (2), 308–322.

Romer, Paul M, "Endogenous technological change," *Journal of political Economy*, 1990, *98* (5, Part 2), S71–S102.

Singh, Jasjit, "Collaborative networks as determinants of knowledge diffusion patterns," *Management science*, 2005, *51* (5), 756–770.

Thompson, Peter and Melanie Fox-Kean, "Patent citations and the geography of knowledge spillovers: A reassessment," *American Economic Review*, 2005, *95* (1), 450–460.

Appendix

A Model

In this appendix section, some of the model derivations that are skipped in the main text and proofs of propositions are provided.

A.1 Derivation of agent HJB equation

Let $\mathcal{U}_i^T(\varepsilon, t)$ denote the life-time utility of type-T agents living in location i, conditional on taste ε. For an infinitesimal time interval of $dt > 0$, the HJB equation can be written in discrete time as follows

$$\mathcal{U}_i^T(\varepsilon, t) = A_i \varepsilon C_i^T(t)dt + \frac{1}{1+\rho dt}\left[\zeta dt \cdot \int \left(\max_j \mathcal{U}_j^T(e_j, t+dt)\right) f_\epsilon(\mathbf{e})d\mathbf{e} + (1-\zeta dt)\cdot \mathcal{U}_i^T(\varepsilon, t+dt)\right] \tag{A.1}$$

The first term in the right hand side of equation (A.1) represents the utility flow due to consumption, amenities and location taste. The second term represents continuation value discounted by time preference parameter ρ. This continuation value is the expected value of drawing a migration shock whose rate is ζ. If agent updates location preferences, then she migrates to the best location for herself in terms of the discounted sum of utility. Otherwise, she stays in the same location.

Using notation defined by equation (3.5), we can organize the terms in (A.1) and take limit $dt \to 0$ to derive the HJB equation in continuous time given by (3.6) in the main text.

A.2 Proof of Proposition 1

We start by taking first order condition of the maximization problem inside (3.9). It reads as

$$W_i^R(t)\,(1-s_i) = \frac{1}{\theta}\alpha_i(t)^{\frac{1}{\theta}} R_i(n,t)^{\frac{1}{\theta}-1} n^{1-\frac{1}{\theta}}\left[\mathcal{V}_i(n+1,t) - \mathcal{V}_i(n,t)\right]$$

Using notation of inventor employment per product line, $r_i(n,t)$, first order condition can be rewritten as

$$W_i^R(t)\,(1-s_i) = \alpha_i(t)^{\frac{1}{\theta}}\frac{1}{\theta} r_i(n,t)^{\frac{1}{\theta}-1}\left[\mathcal{V}_i(n+1,t) - \mathcal{V}_i(n,t)\right] \tag{A.2}$$

I conjecture that the solution to HJB equation (3.9) is $\mathcal{V}_i(n,t) = n v_i(t) Y(t)$ for a function $v_i(t)$. Replacing this conjecture into (A.2) implies that per product line inventor employment is independent of the number of product lines the firm owns, n, such that $r_i(n,t) = r_i(t)$, and equation (3.11) can be derived. Per product line innovation rate can be derived from R&D production function (3.8) substituting $R_i(n,t) = r_i(t)n$. This gives us equation (3.10) in Proposition 1 which states that per product line innovation rate is independent of the firm size proxied by n.

Finally replacing conjecture and first order condition (A.2) into the HJB equation (3.9), and denoting the growth rate of aggregate output as $g(t) \equiv \frac{\dot{Y}(t)}{Y(t)}$, equation (3.12) is derived. This equation governs the time evolution of $v_i(t)$ and states that it is independent of n, as conjectured.

A.3 Proof of Proposition 2

First order condition to the maximization problem (3.14), after substituting $\mathcal{V}_i(1,t) = v_i(t)Y(t)$, can be found as below

$$W_i^R(t)(1-s_i) = \frac{1}{f}\frac{1}{\theta}\alpha_i(t)^{\frac{1}{\theta}}\tilde{r}_i(t)^{\frac{1}{\theta}-1}v_i(t)Y(t) \tag{A.3}$$

Substituting (A.3) into equation (3.14) results in (3.15) as stated by the proposition. Finally we combine incumbent and entrant first order conditions, equations (A.2) and (A.3), respectively. That is, the left hand side of both equations are equal, thus right hands sides have to be equal as well. Organizing terms yields the relationship between entrant and incumbent per product line inventor employments, i.e. equation (3.16). Finally, we can derive equation (3.17) by using R&D production technologies (3.10) and (3.17).

A.4 Evolution of firm size distribution

The mass of product lines owned by firms located in i is denoted by $\psi_i(t)$. To derive the system of equations that govern the time evolution of $\psi_i(t)$, we start with $\mathcal{P}_i(n,t)$ which denotes the measure of firms that are located in i and own n products. $\psi_i(t)$ and $\mathcal{P}_i(n,t)$ are related as follows

$$\psi_i(t) \equiv \sum_{n=1}^{\infty} n\mathcal{P}_i(n,t) \tag{A.4}$$

Unlike Klette and Kortum (2004), the firm size distribution has a spatial angle in the sense that we need to keep track of firm location which is the location where the firm locates its immobile R&D lab at the time of entry.

For $n = 2, 3, \ldots$, we have

$$\dot{\mathcal{P}}_i(n,t) = -n\left[x(t) + z_i(t)\right]\mathcal{P}_i(n,t) + (n-1)z_i(t)\mathcal{P}_i(n-1,t) + (n+1)x(t)\mathcal{P}_i(n+1,t) \tag{A.5}$$

Rate of change in $\mathcal{P}_i(n,t)$ equals three terms. First term represents outflows due to either creative destruction or incumbent firms' own innovation. The second term represents inflows due to innovations of firms with $n-1$ products. The third term also represents inflows to state n due to the fact that firms with $n+1$ products lose one product line as a result of creative destruction.

For state $n = 1$, we have the following equation

$$\dot{\mathcal{P}}_i(1,t) = -\left[x(t) + z_i(t)\right]\mathcal{P}_i(1,t) + \tilde{z}_i(t)\tilde{\psi}_i(t) + 2x(t)\mathcal{P}_i(2,t) \tag{A.6}$$

Similarly, first term represents outflows from the state. Second term stands for rate of entry that takes place in same location. Finally, third term represents inflows from state $n = 2$ due to creative destruction.

While the total mass of product lines is normalized to one, the total mass of firms is endogenous and evolves over time. Denoting by $M_i(t)$ the mass of firms located in i at time t, we can derive the following equation

$$\dot{M}_i(t) = -x(t)\mathcal{P}_i(1,t) + \tilde{z}_i(t)\tilde{\psi}_i(t) \tag{A.7}$$

Rate of change in $M_i(t)$ equals to inflow due to entry in i, the second term, minus outflow due to creative destruction. Note that firms losing their last product line exits the economy.

A.5 Equilibrium production worker wage rate

Demand equation for intermediate good ν from final good producers in a location i is as follows

$$p(\nu,t)k_i(\nu,t) = (1-\beta)Y_i(t)$$

Replacing optimal pricing rule $p(\nu,t) = \lambda a(\nu,t)^{-1}$ yields $k_i(\nu,t) = \lambda^{-1}a(\nu,t)(1-\beta)Y_i(t)$. Replacing this into final good production function and using the constant $\bar{A} = \left(\frac{\lambda}{1-\beta}\right)^{1-\beta}$, we can solve for the output in i as

$$Y_i(t) = \mathcal{A}(t)^{\frac{1-\beta}{\beta}} L_i(t)$$

where $\mathcal{A}(t) = \exp\left(\int_0^1 \log a(\nu,t)d\nu\right)$ is aggregate productivity index of the economy. Then total output equals

$$Y(t) \equiv \sum_i Y_i(t) = \mathcal{A}(t)^{\frac{1-\beta}{\beta}} \bar{L}$$

Demand for labor is given by $W_i^L(t)L_j(t) = \beta Y_i(t)$. Replacing output and solving for wage results in

$$W_i^L(t) = \beta\mathcal{A}(t)^{\frac{1-\beta}{\beta}}$$

Substituting $\mathcal{A}(t)^{\frac{1-\beta}{\beta}}$ with $\frac{Y(t)}{\bar{L}}$ yields equation (3.21) in the main text.

A.6 Growth rate of $\mathcal{A}(t)$

From definition of $\mathcal{A}(t)$, we have

$$\log \mathcal{A}(t) = \int_0^1 \log a\,(\nu,t)\,d\nu$$

Then

$$\frac{\dot{\mathcal{A}}(t)}{\mathcal{A}(t)} = \frac{d\log\mathcal{A}(t)}{dt} = \lim_{dt\to 0}\frac{\log\mathcal{A}(t+dt) - \log\mathcal{A}(t)}{dt}$$

Using definition, we can show

$$\log\mathcal{A}(t+dt) - \log\mathcal{A}(t) = \int_0^1 \left[\log a\,(\nu,t+dt) - \log a\,(\nu,t)\right] d\nu$$

Productivity of a product line ν increases by a proportionality factor of $\lambda > 1$ as a result of creative destruction. The equilibrium rate of creative destruction is $x(t)$. Therefore, for a small time interval of dt, we can write

$$a(\nu, t+dt) = \begin{cases} \lambda a(\nu, t) & \text{with probability } x(t)dt \\ a(\nu, t) & \text{with probability } 1 - x(t)dt \end{cases}$$

Thus, $\log a(\nu, t+dt) - \log a(\nu, t)$ is also a random variable with the following

$$\log a(\nu, t+dt) - \log a(\nu, t) = \begin{cases} \log \lambda & \text{with probability } x(t)dt \\ 0 & \text{with probability } 1 - x(t)dt \end{cases}$$

Integrating over all product lines $\nu \in [0, 1]$, we have

$$\frac{\dot{\mathcal{A}}(t)}{\mathcal{A}(t)} = \lim_{dt \to 0} \frac{\log(\lambda)\, x(t)dt}{dt} = \log(\lambda)\, x(t)$$

as given in the main text.

A.7 Allocation of profits across agents

Let $D(t)$ denote the total profits paid to agents. It equals to sum of profits of intermediate good firms after subtracting R&D costs (including entrants). Then

$$D(t) = \left(1 - \lambda^{-1}\right)(1-\beta)\, Y(t) - \sum_i W_i^R(t)(1 - s_i) R_i(t)$$

From the distribution of profits, we also have

$$D(t) = \sum_i d(t) W_i^L(t) L_i(t) + \sum_i d(t) W_i^R(t) R_i(t)$$

Then solving for $d(t)$ yields

$$d(t) = \frac{\left(1 - \lambda^{-1}\right)(1-\beta)\, Y(t) - \sum_i W_i^R(t)(1 - s_i) R_i(t)}{\sum_i W_i^L(t) L_i(t) + \sum_i W_i^R(t) R_i(t)} \tag{A.8}$$

A.8 Proof of Proposition 3

I prove this proposition under the conjecture that $z_i(t) = z$, $\tilde{z}_i = \tilde{z}$ for all i, and $x(t) = x$ as assumed in the main text. If r_i is constant over time in BGP equilibrium as conjectured, then $\tilde{r}_i$ is also constant satisfying $\tilde{r}_i = \frac{r_i}{F}$ (see Proposition 2). Total inventor employment in a location i equals the sum of inventors in incumbent and entrant firms in that location. Under the conjecture that ψ_i and $\tilde{\psi}_i$ are constant over time in BGP, then $R_i = \psi_i r_i + \tilde{\psi}_i \tilde{r}_i$ is also constant, and

$$\begin{aligned} x &= \sum_i \psi_i z_i + \sum_i \tilde{\psi}_i \tilde{z}_i \\ &= z \sum_i \psi_i + \tilde{z} \sum_i \tilde{\psi}_i \\ &= z + \tilde{z} \end{aligned} \tag{A.9}$$

As $\sum_i \psi_i = \sum_i \tilde{\psi}_i = 1$. System of equations (A.5), (A.6) and (A.7) admit a stationary solution $\mathcal{P}_i(n,t) = \mathcal{P}_i(n)$ such that[17]

$$\mathcal{P}_i(n) = \frac{\tilde{\psi}_i \tilde{z} z^{n-1}}{n x^n}, \qquad n = 1, 2, \ldots \tag{A.10}$$

Then

$$\begin{aligned} \psi_i &= \sum_{n=1}^{\infty} \mathcal{P}_i(n) n = \sum_{n=1}^{\infty} \frac{\tilde{\psi}_i \tilde{z}_i z_i^{n-1}}{n x^n} n = \tilde{\psi}_i \frac{\tilde{z}}{z} \sum_{n=1}^{\infty} \left(\frac{z}{x}\right)^n \\ &= \tilde{\psi}_i \frac{\tilde{z}}{z} \left(\frac{1}{1 - \frac{z}{x}} - 1 \right) \\ &= \tilde{\psi}_i \end{aligned} \tag{A.11}$$

Second line follows from (A.9) given that $z > 0$ and $\tilde{z} > 0$ in BGP equilibrium.

Inventor market clearing implies

$$\begin{aligned} R_i &= \psi_i r_i + \tilde{\psi}_i \tilde{r}_i \\ &= \left(1 + F^{-1}\right) \psi_i r_i \\ &= \frac{1+F}{F} \psi_i \frac{z^\theta}{\alpha_i} \end{aligned}$$

which implies

$$\psi_i = \frac{F}{1+F} z^{-\theta} \alpha_i R_i \tag{A.12}$$

We can solve for z using $\sum_i \psi_i = 1$. That is,

$$\sum_i \psi_i = 1 = \frac{F}{1+F} z^{-\theta} \sum_i \alpha_i R_i$$

which gives

$$z = \left[\frac{F}{1+F} \sum_i \alpha_i R_i \right]^{\frac{1}{\theta}} \tag{A.13}$$

[17] See Klette and Kortum (2004) for details.

Using (A.13) and (A.12), we can show that

$$\psi_i = \tilde{\psi}_i = \frac{\alpha_i R_i}{\sum_i \alpha_i R_i} \tag{A.14}$$

as stated in Proposition 3, satisfying the initial conjecture that ψ_i and $\tilde{\psi}_i$ are time invariant.

A.9 Proof of Proposition 4

From (3.26) and $z_i = z$ for all i, it follows that

$$v_i = v \equiv \frac{\pi}{\rho - g + x - \frac{\theta-1}{\theta} z}$$

First order condition of incumbent firm maximization problem (A.2) and the fact that $\mathcal{V}_i(n,t) = nvY(t)$ in BGP imply that

$$W_i^R(t)(1-s_i) = \alpha_i^{\frac{1}{\theta}} \frac{1}{\theta} r_i^{\frac{1}{\theta}-1} vY(t)$$

in BGP. Defining normalized inventor wage $w_i^R(t) = \frac{W_i^R(t)}{Y(t)}$, and as $z = z_i = (\alpha_i r_i)^{\frac{1}{\theta}}$ for all i, we can rewrite above equation as

$$w_i^R(t)(1-s_i) = z^{1-\theta} \frac{1}{\theta} \alpha_i v$$

for all t. It immediately follows that normalized inventor wage is constant in BGP and equals to

$$w_i^R = \frac{1}{\theta} z^{1-\theta} v \frac{\alpha_i}{1-s_i}$$

as stated in Proposition 4.

A.10 Proof of Proposition 5

In BGP, all consumption rates grow with $g = \frac{1-\beta}{\beta}\log(\lambda)x$. Thus, relative consumptions across locations are time invariant. Let $C_{ij}^T \equiv \frac{C_i^T(t)}{C_j^T(t)}$ for type $T = W, R$. Define normalized value functions $u_i^T(\varepsilon, t) \equiv \frac{\mathcal{U}_i^T(\varepsilon,t)}{C_i^T(t)}$ and $\bar{u}_i^T(t) \equiv \frac{\bar{\mathcal{U}}^T(t)}{C_i^T(t)}$. First we can show that

$$\bar{u}_i^T(t) = \int \max_j \left(u_j^T(e_j, t) C_{ji}^T \right) f_\epsilon(\mathbf{e}) d\mathbf{e} \tag{A.15}$$

Organizing (3.6), we can show that normalized value function $u_i^T(\varepsilon, t)$ satisfies the following functional equation

$$\left[\rho + \zeta - g\right] u_i^T(\varepsilon, t) = A_i \varepsilon + \zeta \bar{u}_i^T(t) + \partial_t u_i^T(\varepsilon, t) \tag{A.16}$$

Following time invariant solutions to normalized agent value functions satisfy the system of equations given by (A.15) and (A.16)

$$u_i^T(\varepsilon) = \frac{A_i\varepsilon + \zeta \bar{u}_i^T}{\rho + \zeta - g} \quad \text{and} \quad \bar{u}_i^T = \int \max_j \left(u_j^T(e_j) C_{ji}^T \right) f_\epsilon(\mathbf{e}) d\mathbf{e}$$

We can recover agent value functions simply using definitions

$$\mathcal{U}_i^T(\varepsilon, t) = \frac{A_i \varepsilon C_i^T(t) + \zeta \bar{\mathcal{U}}^T(t)}{\rho + \zeta - g} \tag{A.17}$$

Replacing value function (A.17) into the migration problem (3.4) yields

$$\max_j \mathcal{U}_j^T(e_j, t) = \frac{\max_j \left(A_j e_j C_j^T(t) \right) + \zeta \bar{\mathcal{U}}^T(t)}{\rho + \zeta - g} \tag{A.18}$$

Substituting (A.18) into the definition of $\bar{\mathcal{U}}^T(t)$, equation (3.5), results in

$$\bar{\mathcal{U}}^T(t) = \frac{1}{\rho + \zeta - g} \left(\mathbb{E}_\epsilon \left[\max_j \left(A_j e_j C_j^T(t) \right) \right] + \zeta \bar{\mathcal{U}}^T(t) \right) \tag{A.19}$$

In order to take expectation in (A.19), we need to derive the distribution of the maximum term. The assumption that locations tastes are independently distributed Frechet, i.e. $\varepsilon_i \sim Frechet(\zeta, 1)$, implies

$$\max_j \left(A_j e_j C_j^T(t) \right) \sim Frechet \left(\zeta, \left[\sum_{j=1}^N \left(A_j C_j^T(t) \right)^\zeta \right]^{\frac{1}{\zeta}} \right) \tag{A.20}$$

Taking expectation in (A.19) and solving for $\bar{U}^T(t)$ yields

$$\bar{\mathcal{U}}^T(t) = \frac{1}{\rho - g} \Gamma \left(1 - \frac{1}{\zeta} \right) \left[\sum_{j=1}^N \left(A_j C_j^T(t) \right)^\zeta \right]^{\frac{1}{\zeta}} \tag{A.21}$$

Finally substituting $\bar{U}^T(t)$ into the agent value function in (A.17) results in

$$\mathcal{U}_i^T(\varepsilon, t) = \frac{A_i \varepsilon C_i^T(t)}{\rho + \zeta - g} + \frac{\zeta}{(\rho + \zeta - g)(\rho - g)} \Gamma \left(1 - \frac{1}{\zeta} \right) \left[\sum_{j=1}^N \left(A_j C_j^T(t) \right)^\zeta \right]^{\frac{1}{\zeta}}$$

as stated in Proposition 5.

A.11 Proof of Proposition 6

Provided the analytical solution for the agent value function from Proposition 5, the migration choice given by (3.4) simplifies such that

$$i^T(t)^\star = \arg\max_j \mathcal{U}_j^T(e_j, t) = \arg\max_j \left(A_j e_j C_j^T(t)\right) \tag{A.22}$$

as the second term on the right hand side of equation (3.29) is independent of locations. Moreover, consumption is proportional to wage rate, i.e.

$$C_j^T(t) = [1 + d - \tau]\, w_i^T Y(t) \tag{A.23}$$

where d is the proportionality factor of profits allocated to agents to wages, given by (A.8), and τ is the labor income tax rate, given by (3.25).[18] As d and τ are independent of locations, replacing (A.23) into the migration problem (A.22) delivers equation (3.30).

In BGP, an agent from i, who has just drawn taste shocks $\{e_m\}_{m=1}^N$, migrates to j if and only if

$$A_j e_j w_j^T \geq A_m e_m C_m^T, \qquad \forall m = 1, \ldots, N$$

Let γ_{ji}^T denote the share of agents of type T in i moving to j among who draw taste shocks. Then,

$$\begin{aligned} \gamma_{ji}^T &= \mathbb{P}\left\{A_j e_j w_j^T \geq A_m e_m w_m^T, \ \forall m \neq j\right\} \\ &= \frac{\left(A_j w_j^T\right)^\xi}{\sum_{m=1}^N \left(A_m w_m^T\right)^\xi}, \qquad \forall i = 1, \ldots, N \end{aligned}$$

The second line follows from the property that taste shocks are distributed Frechet. Notice that γ_{ji}^T does not vary with i. Hence we can assert that $\gamma_{ji}^T = \gamma_j^T$ for all i.

Let M_j^T denote the mass of type-T agents located in j.[19] Then law of motion of this variable depends

[18] In BGP, $d(t)$ is time invariant. In equation (A.8), $Y(t)$ cancels from both numerator and denominator of the expression. As a result, in BGP, we have

$$d = \frac{\left(1 - \lambda^{-1}\right)(1 - \beta) - \sum_i w_i^R (1 - s_i) R_i}{\sum_i w_i^L L_i + \sum_i w_i^R R_i}$$

after a conjecture that worker and researcher populations are stable in BGP. This conjecture holds in BGP as a result of the migration decision given by (A.22) as proven below. Moreover, given same conjecture, we can show that $\tau(t)$ is independent of time in BGP. That is, following from equation (3.25),

$$\tau = \frac{\sum_{i=1}^N s_i w_i^R R_i}{\sum_{i=1}^N w_i^L L_i + w_i^R R_i}$$

[19] $M_j^L = L_j$ and $M_j^R = R_j$.

on migration flows γ_j^T such that

$$\begin{aligned} \dot{M}_j^T &= \underbrace{\sum_{i=1}^{N} \gamma_j^T \zeta M_i^T}_{\text{Inflow}} - \underbrace{\zeta M_j^k}_{\text{Outflow}} \\ &= \zeta \left(\gamma_j^T M^T - M_j^T \right) \end{aligned}$$

where M^T is the total population of type-T, i.e. $M^T = \sum_j M_j^T$. As $\dot{M}_j^T = 0$ in BGP, we have $M_j^T = \gamma_j^T M^T$. That is,

$$\begin{aligned} L_j &= \gamma_j^L \bar{L} \\ R_j &= \gamma_j^R \bar{R} \end{aligned}$$

Therefore, the initial conjecture that population shares are time invariant holds true.

Finally, we can replace the equilibrium wage rates into the expression for γ_j^T. Firstly, $w_j^L = \frac{\beta}{\bar{L}}$ for all j. Hence,

$$\gamma_j^L = \frac{A_j^{\xi}}{\sum_{m=1}^{N} A_m^{\xi}}$$

as given by (3.31). Using equation (4), we can also prove that

$$\gamma_j^R = \frac{A_j^{\xi} \left(\frac{\alpha_j}{1-s_j} \right)^{\xi}}{\sum_{m=1}^{N} A_m^{\xi} \left(\frac{\alpha_m}{1-s_m} \right)^{\xi}}$$

Combining the expressions for γ_j^L and γ_j^R proves equation (3.32) given by Proposition 6.

A.12 Proof of Proposition 7

The expression for z in BGP given by (3.34) is derived in Section A.8 (equation (A.13)).

Total rate of innovation generated in a location i, x_i, equals to the sum of two terms, first of which is the product of mass of product lines owned by incumbent firms located in i and rate of innovation per product line z. The second term is the product of the mass of entrants located in i and the rate of innovation per entrant, i.e. $\tilde{z}$. Thus,

$$x_i = \psi_i z + \tilde{\psi}_i \tilde{z}$$

In Section A.8 equation (A.12), it is shown that $\psi_i = \tilde{\psi}_i = \frac{F}{1+F} z^{-\theta} \alpha_i R_i$. Moreover, we know $\tilde{z} = z/F$.

Thus

$$
\begin{aligned}
x_i &= \left(1 + F^{-1}\right) \psi_i z \\
&= \frac{1+F}{F} \frac{F}{1+F} z^{-\theta} \alpha_i R_i z \\
&= z^{1-\theta} \alpha_i R_i
\end{aligned}
$$

as stated in Proposition 7 equation (3.35).

As shown by equation (A.9), $x = z + \tilde{z} = \frac{1+F}{F} z$. Finally, equation (3.37) follows from equations (3.23) and (3.36).